STAT/MA 41600
In-Class Problem Set #2: August 28, 2015
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Problem Set 2 Answers

1a. Yes, the events are a partition of the sample space. Each outcome has either 0, 1, 2, or 3 bears, so $A_0 \cup A_1 \cup A_2 \cup A_3$ is the whole sample space, and the events A_0, A_1, A_2, A_3 are disjoint, so the events do constitute a sample space.
1b. The probabilities are:

$$P(A_0) = \frac{\binom{3}{0}\binom{6}{5}}{\binom{9}{5}} = \frac{1}{21}; \quad P(A_1) = \frac{\binom{3}{1}\binom{6}{4}}{\binom{9}{5}} = \frac{5}{14}; \quad P(A_2) = \frac{\binom{3}{2}\binom{6}{3}}{\binom{9}{5}} = \frac{10}{21}; \quad P(A_3) = \frac{\binom{3}{3}\binom{6}{2}}{\binom{9}{5}} = \frac{5}{42}.$$

2a. The probability of 0 heads is $(1/2)^{10}$. The probability of 1 head is $(10)(1/2)^{10}$. The probability of 2 heads is $\binom{10}{2}(1/2)^{10}$. So the desired probability is the probability of the complement, i.e., $1 - ((1/2)^{10} + (10)(1/2)^{10} + \binom{10}{2}(1/2)^{10}) = 121/128$.
2b. The probability it takes j flips is $\binom{j-1}{2}(1/2)^j$. So we use the complement to get the desired probability, namely $1 - \sum_{j=3}^{9}\binom{j-1}{2}(1/2)^j = 23/256$.
3a. The probability is $P(A_1 \cup A_2 \cup A_3) = 1/4 + 1/4 + 1/4 - 1/8 - 1/8 - 1/8 + 1/16 = 7/16$.
3b. The events A_1, A_2, A_3 do not constitute a partition of the sample space because the probability of their union is only 7/16, and moreover, the intersections of the events are not empty.
3c. The probability is $P(A_4) = P((A_1 \cup A_2 \cup A_3)^c) = 1 - 7/16 = 9/16$.
3d. The events A_1, A_2, A_3, A_4 do not constitute a partition of the sample space because (even though the probability of their union is 1), the intersections of the events are not empty.
4. There are $4 \times 4 \times 6 = 96$ possible outcomes.
For $3 \leq j \leq 14$, define A_j as the event that the sum of the dice equals j.
Hint: It might be helpful to think about the 7 possible events that are possible with just 2 dice, described here:

x	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

Now we give the number of outcomes in each of the A_j's:

event	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
# of outcomes	1	3	6	10	13	15	15	13	10	6	3	1

So the desired probabilities are:

event	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
# of outcomes	$\frac{1}{96}$	$\frac{3}{96}$	$\frac{6}{96}$	$\frac{10}{96}$	$\frac{13}{96}$	$\frac{15}{96}$	$\frac{15}{96}$	$\frac{13}{96}$	$\frac{10}{96}$	$\frac{6}{96}$	$\frac{3}{96}$	$\frac{1}{96}$