Problem Set 3 Answers

1a. The probability is \((.45 + .42)^5 = (.87)^5 = 0.4984\).

1b. Let each person sampled be a trial. Treat the selection of a female as a good result, selection of a male as a bad result, and selection of a child as neutral. Then the probability that the first adult we interview is a female is \(\frac{45}{45+.42} = 0.5172\).

2a. On a given roll, the probability that a value is 7 or less is \(\frac{21}{36}\). So the probability that we get two results of 8 or higher, followed by a result of 7 or less, is \(\left(\frac{15}{36}\right)^2 \frac{21}{36} = 0.1013\).

2b. The probability that 1 or 2 rolls is sufficient is \(\frac{21}{36} + \left(\frac{15}{36}\right)\left(\frac{21}{36}\right) = 0.8264\). So the probability of the complementary event, i.e., the probability that 3 or more rolls are needed, is \(1 - \frac{119}{144} = \frac{25}{144} = 0.1736\).

2c. We let the sum of the dice be a trial. Then a good trial is exactly a 7, a bad trial is a value (strictly) less than 7, and a neutral trial is (strictly) more than 7. (Notice that we stop when a good or bad trial occurs, i.e., when a roll of 7 or less occurs.) Then the probability of a good trial is \(\frac{6}{36}\) and the probability of a bad trial is \(\frac{15}{36}\). So the desired probability is \(\frac{\frac{6}{36}}{\frac{6}{36} + \frac{15}{36}} = \frac{6}{21} = \frac{2}{7} = 0.2857\).

3. We consider the chosen genre as a trial. A good trial is rock. A bad trial is country, pop, or R&B. A neutral trial is any other genre. Hence, we stop when we get a good or bad trial. So the probability that the person prefers rock is \(\frac{29}{29+11+15+17} = 0.4028\).

4a. We let each of the simultaneous (triples of) rolls of the three dice count as a trial. A good trial has a sum of 5 and the green and blue dice have the same values. A bad trial has a sum of 5 but the green and blue dice do not have the same values. A neutral trial does not have a sum of 5. So a good trial has probability \(P(\{(1,1,3),(2,2,1)\}) = \frac{2}{96}\), and a bad trial has probability \(P(\{(1,3,1),(3,1,1),(2,1,2),(1,2,2)\}) = \frac{4}{96}\). So the desired probability is \(\frac{\frac{2}{96}}{\frac{2}{96} + \frac{4}{96}} = \frac{2}{6} = \frac{1}{3}\).

4b. The solution is exactly the same, even if the dice do not have colors. If you are worried about distinguishing the two 4-sided dice, just put one into your left hand and one into your right hand, and everything proceeds as above.