

Problem Set 8 Answers

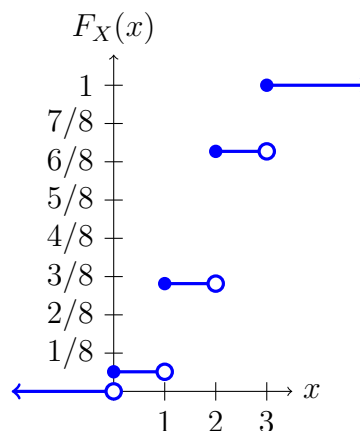
1a. The probability mass function of X is

$$p_X(0) = .4^3 = 0.064; \quad p_X(1) = (3)(.4^2)(.6) = 0.288; \quad p_X(2) = (3)(.4)(.6^2) = 0.432;$$

$$p_X(3) = .6^3 = 0.216; \quad p_X(x) = 0 \text{ otherwise.}$$

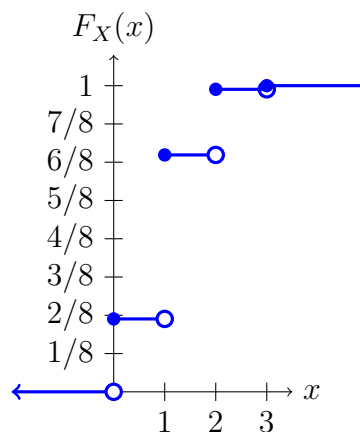
1b. The cumulative distribution function of X is

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.064 & \text{if } 0 \leq x < 1 \\ 0.352 & \text{if } 1 \leq x < 2 \\ 0.784 & \text{if } 2 \leq x < 3 \\ 1 & \text{if } 3 \leq x \end{cases}$$



2. The cumulative distribution function of X is

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 5/21 & \text{if } 0 \leq x < 1 \\ 65/84 & \text{if } 1 \leq x < 2 \\ 83/84 & \text{if } 2 \leq x < 3 \\ 1 & \text{if } 3 \leq x \end{cases}$$



3. To have $X = x$, we must have: (1) the x th role be a value of 3; and (2) the first $x - 1$ roles have exactly 1 value of 3; and (3) the other $x - 2$ out of the first $x - 1$ roles are not values of 3. Therefore $P(X = x) = (x - 1)(1/6)^2(5/6)^{x-2}$ for integers $x \geq 2$.

4a. We have $P(3 \leq X \leq 5) = (\frac{2}{7})(\frac{5}{7})^2 + (\frac{2}{7})(\frac{5}{7})^3 + (\frac{2}{7})(\frac{5}{7})^4 = 5450/16807 = 0.3243$.

4b. We have $P(a \leq X \leq b) = \sum_{x=a}^b (\frac{2}{7})(\frac{5}{7})^{x-1} = (\frac{2}{7})(\frac{5}{7})^{a-1} \sum_{x=0}^{b-a} (\frac{5}{7})^x = (\frac{2}{7})(\frac{5}{7})^{a-1} \frac{(1 - (\frac{5}{7})^{b-a+1})}{(1 - \frac{5}{7})} = (\frac{5}{7})^{a-1} (1 - (\frac{5}{7})^{b-a+1})$.

4c. Yes, this agrees with the answer to 4a in the case $a = 3$ and $b = 5$, because we have $(\frac{5}{7})^{3-1} (1 - (\frac{5}{7})^{5-3+1}) = (\frac{5}{7})^2 (1 - (\frac{5}{7})^3) = 5450/16807 = 0.3243$.