

1. Let Alice roll a 6-sided die and let X denote the result of her roll. Let Bob roll a pair of 4-sided dice and let Y denote the sum of the two values on his two dice. Find $P(X < Y)$.
2. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets, and let X denote the number of red marbles that Bob gets.
 - 2a. Find $p_{X|Y}(0 | 0)$, $p_{X|Y}(1 | 0)$, and $p_{X|Y}(2 | 0)$. Check that these 3 probabilities sum to 1.
 - 2b. Find $p_{X|Y}(0 | 1)$ and $p_{X|Y}(1 | 1)$. Check that these 2 probabilities sum to 1.
3. Consider 5 fish in a bowl: 3 of them are red, and 1 is green, and 1 is blue. Select the fish one at a time, without replacement, until the bowl is empty.

Let $X = 1$ if all of the red fish are selected, before the green fish is selected; and $X = 0$ otherwise.

Let $Y = 1$ if all of the red fish are selected, before the blue fish is selected; and $Y = 0$ otherwise.

 - 3a. Find the joint probability mass function of X and Y .
 - 3b. Make sure that the four probabilities $p_{X,Y}(0, 0)$, $p_{X,Y}(0, 1)$, $p_{X,Y}(1, 0)$, and $p_{X,Y}(1, 1)$ from part 3a have a sum of 1.
 - 3c. Find the probability $p_X(1)$. Find the probability $p_Y(1)$.
 - 3d. Are X and Y independent?
4. Suppose that a person rolls a 6-sided die until the first occurrence of 4 appears, and then the person stops afterwards. Let Y denote the number of rolls that are needed. Let X denote the number of rolls (during this same experiment) on which a value of 3 appears. Find a formula for $p_{X|Y}(x | y)$.