1. Let Alice roll a 6-sided die and let \( X \) denote the result of her roll. Let Bob roll a pair of 4-sided dice and let \( Y \) denote the sum of the two values on his two dice. Find \( P(X < Y) \).

2. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let \( Y \) denote the number of red marbles that Alice gets, and let \( X \) denote the number of red marbles that Bob gets.

   2a. Find \( p_{X|Y}(0 \mid 0) \), \( p_{X|Y}(1 \mid 0) \), and \( p_{X|Y}(2 \mid 0) \). Check that these 3 probabilities sum to 1.

   2b. Find \( p_{X|Y}(0 \mid 1) \) and \( p_{X|Y}(1 \mid 1) \). Check that these 2 probabilities sum to 1.

3. Consider 5 fish in a bowl: 3 of them are red, and 1 is green, and 1 is blue. Select the fish one at a time, without replacement, until the bowl is empty.

   Let \( X = 1 \) if all of the red fish are selected, before the green fish is selected; and \( X = 0 \) otherwise.

   Let \( Y = 1 \) if all of the red fish are selected, before the blue fish is selected; and \( Y = 0 \) otherwise.

   3a. Find the joint probability mass function of \( X \) and \( Y \).

   3b. Make sure that the four probabilities \( p_{X,Y}(0,0) \), \( p_{X,Y}(0,1) \), \( p_{X,Y}(1,0) \), and \( p_{X,Y}(1,1) \) from part 3a have a sum of 1.

   3c. Find the probability \( p_X(1) \). Find the probability \( p_Y(1) \).

   3d. Are \( X \) and \( Y \) independent?

4. Suppose that a person rolls a 6-sided die until the first occurrence of 4 appears, and then the person stops afterwards. Let \( Y \) denote the number of rolls that are needed. Let \( X \) denote the number of rolls (during this same experiment) on which a value of 3 appears. Find a formula for \( p_{X|Y}(x \mid y) \).