STAT/MA 41600
In-Class Problem Set #11: September 18, 2015

1. Suppose that 60% of people in Chicago are fans of da Bears. Assume that the fans' preferences are independent. We interview 3 fans, and we let X denote the number of fans of da Bears. For $i = 1, 2, 3$, let $X_i = 1$ if the ith person is a fan of da Bears, and let $X_i = 0$ otherwise. So we have $X = X_1 + X_2 + X_3$. Find $\mathbb{E}(X_i)$ for $i = 1, 2, 3$, and then find $\mathbb{E}(X)$.

2. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets, and let X denote the number of red marbles that Bob gets.
2a. For $i = 1, 2$, let $Y_i = 1$ if the ith ball that Alice selects is red, and $Y_i = 0$ otherwise. So we have $Y = Y_1 + Y_2$. Find $\mathbb{E}(Y_i)$ for $i = 1, 2$, and then find $\mathbb{E}(Y)$.
2b. Temporarily view one of the red balls as having a #1 painted on it, and the other red ball as having a #2 painted on it. For $i = 1, 2$, let $Z_i = 1$ if the ith red ball is picked by Alice (at any time, i.e., on either of her roles), and $Z_i = 0$ otherwise. So we have $Y = Z_1 + Z_2$. Find $\mathbb{E}(Z_i)$ for $i = 1, 2$, and then find $\mathbb{E}(Y)$.

3. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. Roll both dice, and let X denote the number of red sides that appear. Treat the red/green/blue die as die #1, and the red/blue die as die #2. Let $X_i = 1$ if the ith die is red, or $X_i = 0$ otherwise. So we have $X = X_1 + X_2$. Find $\mathbb{E}(X_i)$ for $i = 1, 2$, and then find $\mathbb{E}(X)$.

4. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). We keep track (in order) of the kind of bears that we get. Let X denote the number of red bears selected.
4a. For $i = 1, 2, 3, 4, 5$, let $X_i = 1$ if the ith bear selected is red, and $X_i = 0$ otherwise. So we have $X = X_1 + X_2 + X_3 + X_4 + X_5$. Find $\mathbb{E}(X_i)$ for $i = 1, 2, 3, 4, 5$, and then find $\mathbb{E}(X)$.
4b. Refer to the red father bear as red bear #1, and the red mother bear as red bear #2, and the red baby bear as red bear #3. For $i = 1, 2, 3$, let $Y_i = 1$ if the ith red bear is selected (at any time, i.e., on any of the five selections), and $Y_i = 0$ otherwise. So we have $X = Y_1 + Y_2 + Y_3$. Find $\mathbb{E}(Y_i)$ for $i = 1, 2, 3$, and then find $\mathbb{E}(X)$.