STAT/MA 41600

In-Class Problem Set #12: September 21, 2015

1. Suppose that 60% of people in Chicago are fans of da Bears. Assume that the fans' preferences are independent. We interview 3 fans, and we let X denote the number of fans of da Bears.

1a. Find $\mathbb{E}(X^2)$ using the probability mass function of X.

1b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2 + X_3$, where the X_j's are independent indicators. Expand $(X_1 + X_2 + X_3)^2$ into 9 terms, where 6 of them will behave one way, and the other 3 will behave another way.

1c. Find $\text{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.

1d. Find $\text{Var}(X)$ in a different manner, namely, by writing $X = X_1 + X_2 + X_3$ for some independent indicators: First find $\text{Var}(X_j)$ for each j, and then use the fact that the variance of the sum of independent random variables equals the sum of the variances.

2. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob also chooses 2 marbles without replacement. Let Y denote the number of red marbles that Alice gets.

2a. Find $\mathbb{E}(Y^2)$ using the probability mass function of Y.

2b. Find $\mathbb{E}(Y^2)$ in a different way, namely, using the fact that $Y = Y_1 + Y_2$, where the Y_j's are **dependent** indicators. Expand $(Y_1 + Y_2)^2$ into 4 terms, where 2 of them will behave one way, and the other 2 will behave another way.

2c. Find $\text{Var}(Y)$ using your answer to $\mathbb{E}(Y^2)$ and your answer to $\mathbb{E}(Y)$ from last week.

3. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. Roll both dice, and let X denote the number of red sides that appear.

3a. Find $\mathbb{E}(X^2)$ using the probability mass function of X.

3b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2$, where the X_j's are independent indicators. Expand $(X_1 + X_2)^2$ into 4 terms, where 2 of them will behave one way, and the other 2 will behave another way.

3c. Find $\text{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.

3d. Find $\text{Var}(X)$ in a different manner, namely, by writing $X = X_1 + X_2$ for some independent indicators: First find $\text{Var}(X_j)$ for each j, and then use the fact that the variance of the sum of independent random variables equals the sum of the variances.

4. Consider a collection of 9 bears. There is a family of red bears consisting of one father bear, one mother bear, and one baby bear. There is a similar green bear family, and a similar blue bear family. We draw 5 consecutive times from this collection *without replacement* (i.e., not returning the bear after each draw). We keep track (in order) of the kind of bears that we get. Let X denote the number of red bears selected. For $i = 1, 2, 3, 4, 5$, let $X_i = 1$ if the ith bear selected is red, and $X_i = 0$ otherwise. Refer to the red father bear as red bear #1, and the red mother bear as red bear #2, and the red baby bear as red bear #3. For $i = 1, 2, 3$, let $Y_i = 1$ if the ith red bear is selected (at any time, i.e., on any of the five selections), and $Y_i = 0$ otherwise.

4a. Find $\mathbb{E}(X^2)$ using the probability mass function of X.

4b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_5$ (the X_j's are **dependent** indicators). Expand $(X_1 + \cdots + X_5)^2$ into 25 terms, where 20 of them will behave one way, and the other 5 will behave another way.

4c. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = Y_1 + Y_2 + Y_3$ (the Y_j's are **dependent** indicators). Expand $(Y_1 + Y_2 + Y_3)^2$ into 9 terms, where 6 of them will behave one way, and the other 3 will behave another way.

4d. Find $\text{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.