STAT/MA 41600
In-Class Problem Set #15: September 23, 2015

1. Suppose X and Y are independent Binomial random variables, each with $n = 3$ and $p = 9/10$.

1a. Find the probability that X and Y are equal, i.e., find $P(X = Y)$.

1b. Find the probability that X is strictly larger than Y, i.e., find $P(X > Y)$.

1c. Find the probability that Y is strictly larger than X, i.e., find $P(Y > X)$.

[Hint: Once you find the value in 1a, you can very easily find the values in 1b and 1c, with almost no effort at all.]

2. Consider a deck of 15 cards containing 5 blue cards, 5 red cards, and 5 green cards. Shuffle the cards and deal all 15 of the cards out, around a circular table, with one card per seat.

2a. A card is called "isolated" if its color does not agree with either of the nearby cards (i.e., if it has a different color than the card to its right and a different color than the card to its left). Let X denote the number of isolated cards. Find $\mathbb{E}(X)$.

2b. A card is called "semi-happy" if its color agrees with exactly one (but not both) of the nearby cards (i.e., if its color agrees with the color of the card on its left or on its right, but not both). Let Y denote the number of semi-happy cards. Find $\mathbb{E}(Y)$.

2c. A card is called "joyous" if its color agrees with both of the nearby cards (i.e., if its color agrees with the color of the card on its left and on its right). Let Z denote the number of joyous cards. Find $\mathbb{E}(Z)$.

[Hint: You answers to a, b, c, must add up to 15.]

3. You randomly choose cookies from a very large container. Assume that 35% of the cookies are chocolate chip and 65% of the cookies are not chocolate. Assume that your selections of cookies are independent, and assume that the container is so large that these percentages do not change with each subsequent draw. (If you prefer, you can just sample the cookies with replacement, but nobody likes to put cookies back!) Let X denote the number of chocolate chip cookies that you get, when you choose 5 cookies from the cookie jar.

3a. Suppose that your brother also chooses 5 cookies, and let Y denote the number of chocolate chip cookies that he gets. Assume that X and Y are independent. Define $W = X + Y$. Is W a Binomial random variable? If so, why?, and what are the parameters n and p for the random variable W? If W is not a Binomial random variable, then why not?

3b. Is $U = 2X - Y$ a Binomial random variable? If so, why?, and what are the parameters n and p for the random variable U? If U is not a Binomial random variable, then why not?

4. Consider a die with 2 red sides, 2 green sides, and 2 blue sides. Roll the die 5 times, and let X denote the number of times that the die has a red result.

Flip a coin 5 times, and let Y denote the number of times that the coin shows "heads."

4a. Find $\mathbb{E}(X - Y)$.

4b. Find $\mathrm{Var}(X - Y)$.