1. At a lunch buffet there are 13 burgers without guacamole and 7 burgers with guacamole. Isabella, Rodrigo, and their two children each blindly reach for a burger.

   1a. If they independently pick at once and (chaotically!) reach for their burger—and all selections are equally likely—this is just like choosing with replacement. Let \( X \) be the number of the people that reach for burgers with guacamole. What are the expected number and variance of \( X \)?

   1b. More realistically, if they take turns, without replacement, and each person draws blindly from the remaining burgers, this is choosing without replacement. Let \( Y \) be the number of the people that get burgers with guacamole. What are the expected number and variance of \( Y \)?

2. Suppose that \( X \) and \( Y \) are independent Hypergeometric random variables that each have parameters \( N = 6, M = 3, \) and \( n = 2 \). What is the probability that \( X \) and \( Y \) are equal, i.e., what is \( P(X = Y) \)?

3a. Suppose that \( X \) is a Hypergeometric random variable with parameters \( N = 50,000, M = 15,000, \) and \( n = 10 \). Write an exact expression for \( P(X = 4) \). You do not need to evaluate the expression.

3b. Now approximate the expression from part 3a.

4. Consider a Binomial random variable \( X \) with parameters \( n \) and \( p \), and consider a Hypergeometric random variable \( Y \) with parameters \( N, M, n \) (the same value of \( n \) as for the Binomial), and suppose that \( p \) and \( M/N \) happen to be the same value.

   4a. If \( n = 1 \), convince yourself that \( P(X = 1) \) and \( P(Y = 1) \) are always the same. Why? Is there an intuitive reason for this?

   4b. If \( n \geq 2 \), which is larger, \( P(X = n) \) or \( P(Y = n) \)? Why?