STAT/MA 41600
In-Class Problem Set #19: October 2, 2015

1. At a lunch buffet there are 13 burgers without guacamole and 7 burgers with guacamole. Isabella, Rodrigo, and their two children each blindly reach for a burger.

1a. If they independently pick at once and (chaotically!) reach for their burger—and all selections are equally likely—this is just like choosing with replacement. Let X be the number of the people that reach for burgers with guacamole. What are the expected number and variance of X?

1b. More realistically, if they take turns, without replacement, and each person draws blindly from the remaining burgers, this is choosing without replacement. Let Y be the number of the people that get burgers with guacamole. What are the expected number and variance of Y?

2. Suppose that X and Y are independent Hypergeometric random variables that each have parameters $N = 6$, $M = 3$, and $n = 2$. What is the probability that X and Y are equal, i.e., what is $P(X = Y)$?

3a. Suppose that X is a Hypergeometric random variable with parameters $N = 50{,}000$, $M = 15{,}000$, and $n = 10$. Write an exact expression for $P(X = 4)$. You do not need to evaluate the expression.

3b. Now approximate the expression from part **3a.**

4. Consider a Binomial random variable X with parameters n and p, and consider a Hypergeometric random variable Y with parameters N, M, n (the same value of n as for the Binomial), and suppose that p and M/N happen to be the same value.

4a. If $n = 1$, convince yourself that $P(X = 1)$ and $P(Y = 1)$ are always the same. Why? Is there an intuitive reason for this?

4b. If $n \geq 2$, which is larger, $P(X = n)$ or $P(Y = n)$? Why?