

STAT/MA 41600  
In-Class Problem Set #19: October 2, 2015  
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**Problem Set 19 Answers**

**1a.** Since  $X$  is Binomial with  $n = 4$  and  $p = 7/20$ , then  $\mathbb{E}(X) = np = (4)(7/20) = 7/5$ , and  $\text{Var}(X) = np(1 - p) = (4)(7/20)(13/20) = 91/100$ .

**1b.** Since  $Y$  is Hypergeometric with  $N = 20$ ,  $M = 7$ , and  $n = 4$ , then we get  $\mathbb{E}(Y) = n(M/N) = (4)(7/20) = 7/5$ , and  $\text{Var}(Y) = n(M/N)(1 - M/N)(N - n)/(N - 1) = (4)(7/20)(1 - 7/20)(20 - 4)/(20 - 1) = 364/475 = 0.7663$ .

**2.** We have  $P(X = Y) = P(X = Y = 0) + P(X = Y = 1) + P(X = Y = 2) = (1/5)^2 + (3/5)^2 + (1/5)^2 = 11/25$ .

**3a.** The exact expression is  $P(X = 4) = \binom{15000}{4} \binom{35000}{6} / \binom{50000}{10} = 0.20013524\dots$  (You did not have to put the decimal value, of course; it is probably way too large for your calculator.)

**3b.** Since  $X$  is approximately Binomial with  $n = 10$  and  $p = M/N = 35000/50000 = 7/10$ , then  $P(X = 4)$  is approximately equal to  $\binom{10}{4} (3/10)^4 (7/10)^6 = 0.20012095\dots$

**4a.** If  $n = 1$ , then  $P(X = 1) = \binom{1}{1} p^1 (1 - p)^{1-1} = p$  and  $P(Y = 1) = M/N$ , so these are the same value. The intuitive reason is that  $X$  corresponds to a sampling of one item with replacement, to see if it is a success, and  $Y$  corresponds to a sampling of one item without replacement, to see if it is a success, but we don't worry about whether or not we are replacing after picking, because we only pick one item to test.

**4b.** We have  $P(X = n) = \binom{n}{n} p^n (1 - p)^{n-n} = p^n$ , which is equal to  $(M/N)^n$ . In contrast,  $P(Y = n) = \left(\frac{M}{N}\right) \left(\frac{M-1}{N-1}\right) \left(\frac{M-2}{N-2}\right) \cdots \left(\frac{M-n+1}{N-n+1}\right) < (M/N)^n$ , so  $P(Y = n) < P(X = n)$ .