STAT/MA 41600
In-Class Problem Set #25: October 16, 2015

1. Suppose that X has probability density function

$$f_X(x) = \begin{cases} x & \text{for } 0 < x < 1, \\ 2 - x & \text{for } 1 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

1a. Find $P(X \leq 3/4)$.

1b. Find $P(X \leq 5/4)$. (Hint: It is not necessary—but it could be easier—to first find the complementary probability.)

1c. Find a formula for the CDF $F_X(x)$. (Hint: It is worthwhile to do this in a piecewise manner, since $f_X(x)$ is defined piecewise. I.e., it is helpful to find $F_X(x)$ for $0 < x < 1$ and then to find $F_X(x)$ for $1 < x < 2$.)

1d. Do your answers to **a** and **b** each agree with your answer to **c**, in the specific cases $x = 3/4$ and $x = 5/4$?

2. Suppose X and Y have a constant joint density on the square with vertices $(0, 0), (4, 0), (4, 4), (0, 4)$.
2a. For $0 < a < 4$, find $P(X + Y \leq a)$.
2b. For $4 < a < 8$, find $P(X + Y \geq a)$. (Then the complement $P(X + Y \leq a)$ is easy.)
2c. If you write $W = X + Y$, the work from **a** and **b** automatically yields an expression for the CDF $F_W(w) = P(W \leq w)$ of W. Differentiate this CDF $F_W(w)$ to find the density $f_W(w)$ of W.

3. Suppose X and Y have joint probability density function

$$f_{X,Y}(x, y) = 21e^{-3x-7y}$$

for $x > 0$ and $y > 0$; and $f_{X,Y}(x, y) = 0$ otherwise.
3a. Compute $P(Y \geq X)$.
3b. Compute $P(Y \leq 3X)$.
3c. Compute $P(Y \geq 1/10)$.

4a. In the setup of question **3**, find the probability density function $f_X(x)$ of X.
4b. In the setup of question **3**, find the probability density function $f_Y(y)$ of Y.
4c. Use your answer to **4b** to find $P(Y \geq 1/10)$. Does your answer agree with your answer to **3c**?