1. Consider a pair of random variables $X$ and $Y$ with joint probability density function $f_{X,Y}(x, y) = \frac{1}{8}xy$ for $x, y$ in the triangle where $0 < x < 2$ and $0 < y < 2x$, and $f_{X,Y}(x, y) = 0$ otherwise.

1a. Are $X$ and $Y$ independent? Why or why not?
1b. Find $P(X \leq 1)$ using the joint density $f_{X,Y}(x, y)$.
1c. Find the density $f_X(x)$.
1d. Use the density $f_X(x)$ to find $P(X \leq 1)$. Does your answer agree with your answer to b?

2. Suppose $X$ and $Y$ have joint density $f_{X,Y}(x, y) = 10e^{-3x-2y}$ for $x, y$ in the region where $0 < x < y$, and $f_{X,Y}(x, y) = 0$ otherwise.

2a. Find $P(Y > 2X)$. (Just a side comment, not a hint: We already know $P(Y > X) = 1$.)
2b. Find the density $f_X(x)$ of $X$.

3. Suppose $X, Y$ has joint density

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{225}(5-x)(6-y) & \text{if } 0 \leq x \leq 5 \text{ and } 0 \leq y \leq 6, \\ 0 & \text{otherwise.} \end{cases}$$

3a. Are $X$ and $Y$ independent? Why or why not?
3b. Find the density $f_X(x)$ of $X$.
3c. Find the density $f_Y(y)$ of $Y$.

4. Suppose $X$ is a continuous random variable with density $f_X(x) = 3e^{-3x}$ for $x > 0$, and $f_X(x) = 0$ otherwise. Suppose $Y$ is a continuous random variable with density $f_Y(y) = 5e^{-5y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Finally, suppose that $X$ and $Y$ are independent. Define $Z$ as the minimum of $X$ and $Y$, i.e., $Z = \min(X, Y)$.

4a. Find the density $f_Z(z)$ of $Z$.
4b. Find $P(Z > 1/10)$. 

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In-Class Problem Set #26: October 19, 2015