STAT/MA 41600

In-Class Problem Set #27: October 21, 2015

1. Suppose X and Y have joint probability density function

$$f_{X,Y}(x,y) = 70e^{-3x-7y}$$

for $0 < x < y$; and $f_{X,Y}(x,y) = 0$ otherwise.

1a. For $x > 0$, find the density $f_X(x)$ of X.

1b. For $x > 0$, use your answer to **a** to find the conditional density $f_{Y|X}(y \mid x)$ of Y, given $X = x$.

1c. When $x = 1/10$, verify that the conditional probability density function $f_{Y|X}(y \mid \frac{1}{10})$ is a valid density, i.e., that (1) it is nonnegative and (2) we get 1 when integrating over the relevant y's.

1d. Find the conditional probability that $Y > 1/4$, given $X = 1/10$, i.e., $P(Y > 1/4 \mid X = 1/10)$.

2a. How do you setup a calculation to compute $P(Y > 1/4 \mid X > 1/10)$? Do you need the conditional probability density function $f_{Y|X}(y \mid x)$ for this calculation? (Notice that we are now conditioning on $X > 1/10$ instead of $X = 1/10$.) Go ahead and calculate $P(Y > 1/4 \mid X > 1/10)$. It might help to draw separate pictures for the numerator and denominator, so that you get the regions of integration right.

2b. Find the conditional probability that $Y < 1/3$, given $X > 1/10$, i.e., $P(Y < 1/3 \mid X > 1/10)$.

3. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0,0)$, $(2,0)$, and $(0,8)$.

3a. For $0 \le x \le 2$, find the conditional density $f_{Y|X}(y \mid x)$ of Y, given $X = x$.

3b. Find the conditional probability that $Y \le 4$, given $X = 1/2$. I.e., find $P(Y \le 4 \mid X = 1/2)$.

3c. Find the conditional probability that $Y \le 4$, given $X \le 1/2$. I.e., find $P(Y \le 4 \mid X \le 1/2)$.

4a. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0,0)$, $(5,0)$, and $(0,5)$. For a (fixed) value of x with $0 \le x \le 5$, find the conditional density $f_{Y|X}(y \mid x)$ of Y, given $X = x$.

4b. Can you generalize this? Suppose that $c > 0$ is a fixed constant. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0,0)$, $(c,0)$, and $(0,c)$. For a (fixed) value of x with $0 \le x \le c$, find the conditional density $f_{Y|X}(y \mid x)$ of Y, given $X = x$.