1. Suppose $X$ and $Y$ have joint probability density function
\[ f_{X,Y}(x, y) = 70e^{-3x-7y} \]
for $0 < x < y$; and $f_{X,Y}(x, y) = 0$ otherwise.

1a. For $x > 0$, find the density $f_X(x)$ of $X$.

1b. For $x > 0$, use your answer to a to find the conditional density $f_{Y|X}(y|x)$ of $Y$, given $X = x$.

1c. When $x = 1/10$, verify that the conditional probability density function $f_{Y|X}(y|\frac{1}{10})$ is a valid density, i.e., that (1) it is nonnegative and (2) we get 1 when integrating over the relevant $y$'s.

1d. Find the conditional probability that $Y > 1/4$, given $X = 1/10$, i.e., $P(Y > 1/4 | X = 1/10)$.

2a. How do you setup a calculation to compute $P(Y > 1/4 | X > 1/10)$? Do you need the conditional probability density function $f_{Y|X}(y|x)$ for this calculation? (Notice that we are now conditioning on $X > 1/10$ instead of $X = 1/10$.) Go ahead and calculate $P(Y > 1/4 | X > 1/10)$.

2b. Find the conditional probability that $Y < 1/3$, given $X > 1/10$, i.e., $P(Y < 1/3 | X > 1/10)$.

3. Consider a pair of random variables $X, Y$ with constant joint density on the triangle with vertices at $(0,0), (2,0),$ and $(0,8)$.

3a. For $0 \leq x \leq 2$, find the conditional density $f_{Y|X}(y|x)$ of $Y$, given $X = x$.

3b. Find the conditional probability that $Y \leq 4$, given $X = 1/2$. I.e., find $P(Y \leq 4 | X = 1/2)$.

3c. Find the conditional probability that $Y \leq 4$, given $X \leq 1/2$. I.e., find $P(Y \leq 4 | X \leq 1/2)$.

4a. Consider a pair of random variables $X, Y$ with constant joint density on the triangle with vertices at $(0,0), (5,0),$ and $(0,5)$. For a (fixed) value of $x$ with $0 \leq x \leq 5$, find the conditional density $f_{Y|X}(y|x)$ of $Y$, given $X = x$.

4b. Can you generalize this? Suppose that $c > 0$ is a fixed constant. Consider a pair of random variables $X, Y$ with constant joint density on the triangle with vertices at $(0,0), (c,0),$ and $(0,c)$. For a (fixed) value of $x$ with $0 \leq x \leq c$, find the conditional density $f_{Y|X}(y|x)$ of $Y$, given $X = x$. 