STAT/MA 41600
In-Class Problem Set #28: October 23, 2015
Solutions by Mark Daniel Ward

1. One method is that we can compute

$$\mathbb{E}(X) = \int_0^\infty \int_x^\infty (x)(70e^{-3x-7y})\,dy\,dx = \int_0^\infty (x)(70e^{-3x}) \int_x^\infty e^{-7y}\,dy\,dx = \int_0^\infty (x)(70e^{-3x})(1/7)e^{-7x}\,dx,$$

which simplifies to

$$\mathbb{E}(X) = \int_0^\infty (x)(10e^{-10x})\,dx = 1/10.$$

FYI, if you decided (instead) to just directly use the density of X, namely, $f_X(x) = 10e^{-10x}$ for $x > 0$, we get exactly the line above, $\mathbb{E}(X) = \int_0^\infty (x)(10e^{-10x})\,dx = 1/10$.

2. One method is that we can compute

$$\mathbb{E}(Y) = \int_0^\infty \int_x^\infty (y)(70e^{-3x-7y})\,dy\,dx = \int_0^\infty (70e^{-3x}) \int_x^\infty ye^{-7y}\,dy\,dx = \int_0^\infty (70e^{-3x})\frac{7x+1}{49}e^{-7x}\,dx,$$

which simplifies to

$$\mathbb{E}(Y) = \int_0^\infty (70/49)(7x+1)e^{-10x}\,dx = 17/70.$$

A second method is that we can compute

$$\mathbb{E}(Y) = \int_0^\infty \int_0^y (y)(70e^{-3x-7y})\,dx\,dy = \int_0^\infty (y)(70e^{-7y}) \int_0^y e^{-3x}\,dx\,dy = \int_0^\infty (y)(70e^{-7y})(1/3)(1-e^{-3y})\,dy,$$

which simplifies to

$$\mathbb{E}(Y) = \int_0^\infty (y)(70/3)(e^{-7y} - e^{-10y})\,dy = 10/21 - 7/30 = 17/70.$$

3. One method is that we can compute

$$\mathbb{E}(X) = \int_0^2 \int_0^{8-4x} (x)(1/8)\,dy\,dx = \int_0^2 (x)(1/8) \int_0^{8-4x} 1\,dy\,dx = \int_0^2 (x)(1/8)(8-4x)\,dx,$$

which simplifies to

$$\mathbb{E}(X) = \int_0^2 (x)\Big(\frac{8-4x}{8}\Big)\,dx = 2/3.$$

FYI, if you decided (instead) to just directly use the density of X, namely, $f_X(x) = \frac{8-4x}{8}$ for $0 \le x \le 2$, we get exactly the line above, $\mathbb{E}(X) = \int_0^2 (x)(\frac{8-4x}{8})\,dx = 2/3$.

4a. We have

$$\mathbb{E}(Y) = \int_0^\infty (y)(5e^{-5y})\,dy = (y)(-e^{-5y})\Big|_{y=0}^\infty - \int_0^\infty -e^{-5y}\,dy = -(1/5)e^{-5y}\Big|_{y=0}^\infty = 1/5.$$

4b. We have

$$\mathbb{E}(Y) = \int_0^\infty (y)(\lambda e^{-\lambda y})\,dy = (y)(-e^{-\lambda y})\Big|_{y=0}^\infty - \int_0^\infty -e^{-\lambda y}\,dy = -(1/\lambda)e^{-\lambda y}\Big|_{y=0}^\infty = 1/\lambda.$$