

STAT/MA 41600
In-Class Problem Set #29: October 26, 2015

1. Suppose X and Y have joint probability density function

$$f_{X,Y}(x,y) = 70e^{-3x-7y}$$

for $0 < x < y$; and $f_{X,Y}(x,y) = 0$ otherwise.

1a. Find $\mathbb{E}(X^2)$. (You may either use the joint density given here, or the density $f_X(x)$ that was found in **1a** of the Problem Set 27.)

1b. Find $\text{Var}(X)$. (You already have $\mathbb{E}(X)$ from Problem Set 28.)

2. Consider a pair of random variables X, Y with constant joint density on the rectangle with vertices $(0, 0)$, $(5, 0)$, $(5, 8)$, $(0, 8)$.

2a. Find $\mathbb{E}(XY)$.

2b. Are X and Y independent?

Now use **2c** and **2d** to double-check your solution to **2a**:

2c. Find $\mathbb{E}(X)$.

2d. Find $\mathbb{E}(Y)$.

3. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0, 0)$, $(2, 0)$, and $(0, 8)$.

3a. Find $\mathbb{E}(X^2)$.

3b. Find $\mathbb{E}(XY)$.

- 4a. Suppose that Y is an exponential random variable with probability density function $f_Y(y) = 5e^{-5y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise.

4a. Compute $\mathbb{E}(Y^2)$.

4b. Compute $\text{Var}(Y)$. (You already have $\mathbb{E}(Y)$ from the previous problem set.)