STAT/MA 41600

In-Class Problem Set #32: October 30, 2015

1. On Halloween you are visiting a haunted corn maze. Let X denote the time (in minutes) until you hear someone scream. Let Y denote the time (in minutes) until you get scared by a ghost. Suppose that X and Y are independent Exponential random variables, with $\mathbb{E}(X) = 1/2$ and $\mathbb{E}(Y) = 1/3$. Find the probability that, within the next 1 minute, you don't hear anybody scream and you don't get scared by a ghost. In other words, find $P(\min(X, Y) > 1)$, or equivalently, $P(X > 1 \;\&\; Y > 1)$.

2a. In the scenario from question **1**, start listening for screams at 11:56 PM. Given that nobody has screamed by 11:59 PM, use integration to compute the conditional probability that nobody screams by midnight. I.e., find $P(X > 4 \mid X > 3)$.

2b. Does your solution agree with what you would find, if you had (instead) just used the memoryless property of Exponential random variables? In other words, was your conditional probability equal to $P(X > 1)$?

2c. To convince yourself that the memoryless property is something special for Exponential random variables, we could demonstrate that it does not hold for other kinds of random variables. For instance, if U is a Continuous Uniform random variable on $[0, 10]$, show that $P(U > 4 \mid U > 3)$ is not equal to $P(U > 1)$.

3a. Suppose that there are five bats resting in a row on top of a scary barn, at this haunted corn maze. You number them from 1 to 5, and you use $X_1, \ldots, X_5$ to denote the times (in minutes) until the bats fly away from the barn. Suppose $X_1, \ldots, X_5$ are independent Exponential random variables, with $\mathbb{E}(X_j) = 8$. Find the probability that none of the bats depart during the next 20 minutes, i.e., $P(\min(X_1, \ldots, X_5) \geq 20)$, i.e., $P(X_j \geq 20$ for all $j)$.

If you prefer, you can find the probability that at least one of the bats departs during the next 20 minutes, i.e., $P(\max(X_1, \ldots, X_5) \geq 20)$, i.e., $P(X_j \geq 20$ for at least one $j)$.

3b. Eventually you leave the scary barn, and you walk over to the haunted mansion. Everything seems eerily quiet (almost too quiet!). Then your blood runs cold...and your spine begins to shiver...as you realize that there are n bats perched on the roof of the haunted mansion. As in **3a**, number the bats, and use $Y_1, \ldots, Y_n$ to denote the times (in minutes) until the bats fly away from the barn. Suppose $Y_1, \ldots, Y_n$ are independent Exponential random variables, with $\mathbb{E}(Y_j) = 8$. Find the probability that none of these n bats depart during the next 20 minutes, i.e., $P(\min(Y_1, \ldots, Y_n) \geq 20)$, i.e., $P(X_j \geq 20$ for all $j)$.

If you prefer, you can find the probability that at least one of the bats departs during the next 20 minutes, i.e., $P(\max(Y_1, \ldots, Y_n) \geq 20)$, i.e., $P(Y_j \geq 20$ for at least one $j)$.

4. Finally it is almost 2 AM and you are scared and tired. You think it is time to leave the haunted corn maze, but none of your friends are around, when you get back to your car. You came to the corn maze with two friends, Alejandro and Brenda. Let X denote the time (in minutes) until Alejandro arrives at the car and let Y denote the time (in minutes) until Brenda arrives at the car. Since you and Alejandro and Brenda all got hopelessly lost from each other during the night at the corn maze, you can assume X and Y are independent Exponential random variables, which each have mean 5, i.e., $\mathbb{E}(X) = \mathbb{E}(Y) = 5$.

Find $P(X < \frac{1}{2}Y)$, i.e., find the probability that your waiting time for Alejandro is less than half of your waiting time for Brenda.

[[Hint: If you prefer, equivalently, you can find $P(2X < Y)$. Just draw the region where $2X < Y$ in the plane, and then integrate the joint density of X and Y over that region.]]

P.S. Dr Ward hopes that you all have a safe and enjoyable Halloween weekend!