

STAT/MA 41600
In-Class Problem Set #32 part 2: November 2, 2015

1. Suppose that you wait for the telephone to ring, and your waiting time X is an Exponential random variable with average 5 minutes. The first time the phone rings, it is a wrong number, so the conversation takes no time at all, and you hang up and then immediately begin waiting again for the next time the phone rings. This second waiting time Y (after the hangup occurs) is also an Exponential random variable with average 5 minutes. Also assume that Y is independent of X .

1a. Find $P(X + Y < 12)$.

1b. If $a > 0$ is fixed, find an expression for $P(X + Y < a)$.

1c. Does your answer to **1a** agree with your expression in **1b**? I.e., if you plug in $a = 12$ to your answer in **1b**, does it agree with your answer to **1a**?

2a. Suppose that U is a Continuous Uniform random variable with constant density in the interval $(0, 1)$. Define $X = -\frac{1}{5} \ln U$.

2a. What values can X take on?

2b. What is the CDF of X ?

2c. What kind of random variable is X ?

3. Consider independent exponential random variables X and Y that each have average 1. Find $P(|X - Y| > 1)$.

[Hint: Equivalently, find $P(X - Y > 1 \text{ or } Y - X > 1)$, which is $P(X - Y > 1) + P(Y - X > 1)$, since the corresponding regions are disjoint, i.e., are nonoverlapping.]

4. Elena just got engaged to be married. She posts a message about the engagement on Facebook. Three of her friends, Alicia, Barbara, and Charlene, will click “like” on her post. Use X , Y , and Z (respectively) to denote the waiting times until Alicia, Barbara, and Charlene click “like” on this post, and assume that these three random variables are independent. Assume each of the random variables is an Exponential random variable that has an average of 2 minutes.

4a. Find $P(X < 1)$.

4b. Use your answer to **4a** to find the probability that all 3 friends “like” the post within 1 minute.

4c. Use your answer to **4a** to find the probability that none of the 3 friends “like” the post within 1 minute.

4d. Use your answer to **4a** to find the probability that exactly 1 of the 3 friends “likes” the post within 1 minute.

4e. Use your answer to **4a** to find the probability that exactly 2 of the 3 friends “like” the post within 1 minute.

4f. Let V denote the number of friends (among these 3) who “like” the post within 1 minute. Then V is a discrete random variable. What kind of random variable is V ? [Hint: In **4b**, we have $P(V = 3)$; in **4c**, we have $P(V = 0)$; in **4d**, we have $P(V = 1)$; in **4e**, we have $P(V = 2)$. Your answers in **4b**, **4c**, **4d**, **4e** should sum to 1.]