1a. We have \( P(X_1 + X_2 < a) = \int_0^a \int_0^{a-x_1} \lambda e^{-\lambda x_1} \lambda e^{-\lambda x_2} \, dx_2 \, dx_1 \). For the inner integral, we focus on the \( x_2 \) part of the integrand, and we get: \( \int_0^{a-x_1} \lambda e^{-\lambda x_2} \, dx_2 = 1 - e^{-\lambda a + \lambda x_1} \). So then we have \( P(X_1 + X_2 < a) = \int_0^a \lambda e^{-\lambda x_1} (1 - e^{-\lambda a + \lambda x_1}) \, dx_1 = \int_0^a \lambda e^{-\lambda x_1} \, dx_1 - \int_0^a \lambda e^{-\lambda a} \, dx_1 = 1 - e^{-\lambda a} - \lambda e^{-\lambda a} = 1 - (1 + \lambda a)e^{-\lambda a} \).

1b. We have \( P(X_1 + X_2 + X_3 < a) = \int_0^a \int_0^{a-x_1} \int_0^{a-x_1-x_2} \lambda e^{-\lambda x_1} \lambda e^{-\lambda x_2} \lambda e^{-\lambda x_3} \, dx_3 \, dx_2 \, dx_1 \). [You do not have to calculate this triple integral, but just FYI, \( P(X_1 + X_2 + X_3 < a) = 1 - e^{-\lambda a} (1 + \lambda a + (\lambda a)^2) \).]

2a. Yes! The random variables \( U \) and \( V \) are independent, because \( f_{U,V}(u,v) \) can be factored into \( u \) and \( v \) parts.

2b. By symmetry, we have \( f_U(u) = 3e^{-3u} \) for \( u \) positive, and \( f_U(u) = 0 \) otherwise.

2c. The random variable \( X \) is a Gamma random variable with parameters \( r = 2 \) and \( \lambda = 3 \), so the density of \( X \) is \( f_X(x) = 9xe^{-3x} \) for \( x > 0 \), and \( f_X(x) = 0 \) otherwise.

2d. We have \( P(X \leq 1/2) = F_X(1/2) = 1 - e^{-(3)(1/2)}(1 + (3)(1/2)) = 1 - (5/2)e^{-3/2} \). Or we could calculate \( P(X \leq 1/2) = \int_0^{1/2} 9xe^{-3x} \, dx = 1 - (5/2)e^{-3/2} \).

3a. We have \( P(U > V) + P(U = V) + P(U < V) = 1 \), and \( P(U = V) = 0 \), so it must be the case that \( P(U > V) = 1/2 \).

3b. The random variable \( Y \) is a Binomial random variable with parameters \( n = 2 \) and \( p = 1 - e^{-(3)(1/10)} = 0.2592 \).

4a. The random variable \( W \) is a Gamma random variable with \( r = 3 \) and \( \lambda = 1/5 \). The density of \( W \) is \( f_W(w) = \frac{1/5}2 w^2 e^{-(1/5)w} \) for \( w > 0 \), and \( f_W(w) = 0 \) otherwise.

4b. The variance of \( W \) is \( \text{Var}(W) = r/\lambda^2 = \frac3{(1/5)^2} = 75 \).

4c. The variance of \( W/60 \) is \( \text{Var}(W/60) = 75(1/60)^2 = 1/48 \).