STAT/MA 41600
In-Class Problem Set #34: November 6, 2015
Solutions by Mark Daniel Ward

1a. Using the formula for the expected value of a Beta random variable, we have $\mathbb{E}(X) = \alpha/(\alpha+\beta) = \frac{8}{8+2} = \frac{8}{10}$; or, if you prefer to calculate: $\mathbb{E}(X) = \int_0^1 (x)\frac{9!}{7!1!}x^7(1-x)^1\,dx = 4/5$.

1b. We see that $f_X(x) = \frac{9!}{7!1!}x^7(1-x)^1$ for $0 \le x \le 1$, and $f_X(x) = 0$ otherwise.

1c. Yes! The function $f_X(x)$ is always nonnegative, and we have $\int_0^1 \frac{9!}{7!1!}x^7(1-x)^1\,dx = 1$.

2a. We have $P(X > 0.90) = \int_{0.90}^1 \frac{9!}{7!1!}x^7(1-x)^1\,dx = 0.2252$.

2b. We have $P(X > 0.90 \mid X > 0.80) = \frac{P(X>0.90\ \&\ X>0.80)}{P(X>0.80)} = \frac{P(X>0.90)}{P(X>0.80)}$. The numerator, as in part a, is 0.2252. The denominator is $\int_{0.80}^1 \frac{9!}{7!1!}x^7(1-x)^1\,dx = 0.5638$. Putting these results together, the conditional probability is $P(X > 0.90 \mid X > 0.80) = \frac{0.2252}{0.5638} = 0.3994$.

3. We have $P(X < 0.15) = \int_0^{0.15} \frac{21!}{1!19!}x^1(1-x)^{19}\,dx = \int_{0.85}^1 \frac{21!}{1!19!}(1-u)^1u^{19}\,du = 0.8450$.

4a. No! The sum of two independent Bernoulli random variables (with the same parameters p) is a Binomial random variable with parameters $n = 2$ and p.

4b. Yes! The sum of two independent Binomial random variables (with the same parameters p) is a Binomial random variable too. The value of n is the sum of the values of the n's from the two original Binomial random variables. The value of p is the same as for those original Binomial random variables.

4c. No! The sum of two independent Geometric random variables (with the same parameters p) is a Negative Binomial random variable with parameters $r = 2$ and p.

4d. Yes! The sum of two independent Negative Binomial random variables (with the same parameters p) is a Negative Binomial random variable too. The value of r is the sum of the values of the r's from the two original Negative Binomial random variables. The value of p is the same as for those original Negative Binomial random variables.

4e. Yes! The sum of two independent Poisson random variables is a Poisson random variable too. The value of λ is the sum of the values of the λ's from the two Poisson random variables.

4f. No! The sum of two independent Exponential random variables (with the same parameters λ) is a Gamma random variable with parameters $r = 2$ and λ.

4g. Yes! The sum of two independent Gamma random variables (with the same parameters λ) is a Gamma random variable too. The value of r is the sum of the values of the r's from the two original Gamma random variables. The value of λ is the same as for those original Gamma random variables.