1a. Using the formula for the expected value of a Beta random variable, we have \( \mathbb{E}(X) = \frac{\alpha}{\alpha + \beta} = \frac{8}{8 + 2} = \frac{8}{10} \); or, if you prefer to calculate: \( \mathbb{E}(X) = \int_0^1 (x) \frac{9!}{7!1!} x^7(1-x)^1 \, dx = \frac{4}{5} \).

1b. We see that \( f_X(x) = \frac{9!}{7!1!} x^7(1-x)^1 \) for \( 0 \leq x \leq 1 \), and \( f_X(x) = 0 \) otherwise.

1c. Yes! The function \( f_X(x) \) is always nonnegative, and we have \( \int_0^1 \frac{9!}{7!1!} x^7(1-x)^1 \, dx = 1 \).

2a. We have \( P(X > 0.90) = \int_{0.90}^1 \frac{9!}{7!1!} x^7(1-x)^1 \, dx = 0.2252 \).

2b. We have \( P(X > 0.90 \mid X > 0.80) = \frac{P(X > 0.90 \& X > 0.80)}{P(X > 0.80)} = \frac{P(X > 0.90)}{P(X > 0.80)} \). The numerator, as in part a, is 0.2252. The denominator is \( \int_{0.80}^1 \frac{9!}{7!1!} x^7(1-x)^1 \, dx = 0.5638 \). Putting these results together, the conditional probability is \( P(X > 0.90 \mid X > 0.80) = \frac{0.2252}{0.5638} = 0.3994 \).

3. We have \( P(X < 0.15) = \int_0^{0.15} \frac{21!}{11!9!} x^9(1-x)^9 \, dx = \int_{0.85}^1 \frac{21!}{11!9!} (1-u)^9u^9 \, du = 0.8450 \).

4a. No! The sum of two independent Bernoulli random variables (with the same parameters \( p \)) is a Binomial random variable with parameters \( n = 2 \) and \( p \).

4b. Yes! The sum of two independent Binomial random variables (with the same parameters \( p \)) is a Binomial random variable too. The value of \( n \) is the sum of the values of the \( n \)'s from the two original Binomial random variables. The value of \( p \) is the same as for those original Binomial random variables.

4c. No! The sum of two independent Geometric random variables (with the same parameters \( p \)) is a Negative Binomial random variable with parameters \( r = 2 \) and \( p \).

4d. Yes! The sum of two independent Negative Binomial random variables (with the same parameters \( p \)) is a Negative Binomial random variable too. The value of \( r \) is the sum of the values of the \( r \)'s from the two original Negative Binomial random variables. The value of \( p \) is the same as for those original Negative Binomial random variables.

4e. Yes! The sum of two independent Poisson random variables is a Poisson random variable too. The value of \( \lambda \) is the sum of the values of the \( \lambda \)'s from the two Poisson random variables.

4f. No! The sum of two independent Exponential random variables (with the same parameters \( \lambda \)) is a Gamma random variable with parameters \( r = 2 \) and \( \lambda \).

4g. Yes! The sum of two independent Gamma random variables (with the same parameters \( \lambda \)) is a Gamma random variable too. The value of \( r \) is the sum of the values of the \( r \)'s from the two original Gamma random variables. The value of \( \lambda \) is the same as for those original Gamma random variables.