

STAT/MA 41600
In-Class Problem Set #36: November 11, 2015

1. Consider 40 students in a Chemistry class that each have random amounts of liquid in their beakers. Suppose that the amount of liquid in each beaker is Normally distributed with mean 0.20 liters and standard deviation 0.05 liters.

1a. Find the probability that the class has between 7.9 and 8.1 liters of liquid altogether.

1b. Find the value of b such that there is a 95 percent chance that total quantity of liquid is between $8 - b$ and $8 + b$ liters altogether.

2. Consider 5000 stones whose weights are Normally distributed, each weight having expected value 70 grams, and standard deviation of 8 grams. Let μ denote the expected weight of the entire collection of stones, and let σ^2 denote the variance of the entire collection of stones.

2a. Find the probability that the total weight of the stones exceeds 349000 grams.

2b. Find the probability that the total weight of the stones is within 500 grams of the expected value μ , i.e., if X_1, \dots, X_{5000} are the individual weights, then calculate the probability $P(|X_1 + \dots + X_{5000} - \mu| \leq 500)$, i.e., $P(\mu - 500 \leq X_1 + \dots + X_{5000} \leq \mu + 500)$.

3. Suppose that the heights of blades of grass are Normally distributed, with each height having expected value 4 inches and standard deviation 0.75 inches. Also suppose that the heights are independent.

3a. When ten blades of grass are randomly selected, find the probability that the average height of these blades of grass is between 3.5 and 4.5 inches, i.e., $P(3.5 \leq \frac{X_1 + \dots + X_{10}}{10} \leq 4.5)$.

3b. When ten blades of grass are randomly selected, find the value of “ a ” such that the probability is 0.90 that the average height of these blades of grass is between $4 - a$ and $4 + a$ inches, i.e., $P(4 - a \leq \frac{X_1 + \dots + X_{10}}{10} \leq 4 + a) = 0.90$.

4. Consider the weights of 5 encyclopedia books and 20 novels. The weight of each encyclopedia book is Normally distributed with mean 6 pounds and standard deviation 0.8 pounds. The weight of each novel is Normally distributed with mean 1.4 pounds and standard deviation 0.3 pounds. All of the weights are assumed to be independent.

4a. Find the probability that the total weight of the books does not exceed 60 pounds.

4b. Find the probability that the total weight of the books is between 58 and 62 pounds. (Note: The average weight is not 60 pounds, so this problem is not symmetric around 60.)