

STAT/MA 41600
 In-Class Problem Set #36: November 11, 2015
 Solutions by Mark Daniel Ward

1a. Use X_1, \dots, X_{40} to denote the 40 liquid amounts, so $P(7.9 \leq X_1 + \dots + X_{40} \leq 8.1) = P\left(\frac{7.9-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{X_1+\dots+X_{40}-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{8.1-40(0.20)}{\sqrt{40(0.05)^2}}\right) = P(-0.32 \leq Z \leq 0.32) = 0.6255 - (1 - 0.6255) = 0.2510$.

1b. We have $0.95 = P(8-b \leq X_1 + \dots + X_{40} \leq 8+b) = P\left(\frac{8-b-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{X_1+\dots+X_{40}-40(0.20)}{\sqrt{40(0.05)^2}} \leq \frac{8+b-40(0.20)}{\sqrt{40(0.05)^2}}\right) = P\left(-\frac{b}{0.32} \leq Z \leq \frac{b}{0.32}\right) = P\left(Z \leq \frac{b}{0.32}\right) - (1 - P\left(Z \leq \frac{b}{0.32}\right)) = 2P\left(Z \leq \frac{b}{0.32}\right) - 1$. So $P\left(Z \leq \frac{b}{0.32}\right) = 0.975$. Thus $\frac{b}{0.32} = 1.96$. So we get $b = (0.32)(1.96) = 0.6272$.

2a. Use X_1, \dots, X_{5000} to denote the weights of the stones, so $P(349000 \leq X_1 + \dots + X_{5000}) = P\left(\frac{349000-5000(70)}{\sqrt{5000(8)^2}} \leq \frac{X_1+\dots+X_{5000}-5000(70)}{\sqrt{5000(8)^2}}\right) = P(-1.77 \leq Z) = P(1.77 \geq Z) = 0.9616$.

2b. We have $P(\mu-500 \leq X_1 + \dots + X_{5000} \leq \mu+500) = P\left(\frac{\mu-500-5000(70)}{\sqrt{5000(8)^2}} \leq \frac{X_1+\dots+X_{5000}-5000(70)}{\sqrt{5000(8)^2}} \leq \frac{\mu+500-5000(70)}{\sqrt{5000(8)^2}}\right) = P\left(-\frac{500}{565.69} \leq Z \leq \frac{500}{565.69}\right) = P(-0.88 \leq Z \leq 0.88) = P(Z \leq 0.88) - (1 - P(Z \leq 0.88)) = 0.8106 - (1 - 0.8106) = 0.6212$.

3a. We have $P(3.5 \leq \frac{X_1+\dots+X_{10}}{10} \leq 4.5) = P((3.5)(10) \leq X_1 + \dots + X_{10} \leq (4.5)(10)) = P\left(\frac{(3.5)(10)-10(4)}{\sqrt{10(0.75)^2}} \leq \frac{X_1+\dots+X_{10}-10(4)}{\sqrt{10(0.75)^2}} \leq \frac{(4.5)(10)-10(4)}{\sqrt{10(0.75)^2}}\right) = P(-2.11 \leq Z \leq 2.11) = 0.9826 - (1 - 0.9826) = 0.9652$.

3b. We have $0.90 = P(4-a \leq \frac{X_1+\dots+X_{10}}{10} \leq 4+a) = P((4-a)(10) \leq X_1 + \dots + X_{10} \leq (4+a)(10)) = P\left(\frac{(4-a)(10)-10(4)}{\sqrt{10(0.75)^2}} \leq \frac{X_1+\dots+X_{10}-10(4)}{\sqrt{10(0.75)^2}} \leq \frac{(4+a)(10)-10(4)}{\sqrt{10(0.75)^2}}\right) = P\left(-\frac{10a}{2.37} \leq Z \leq \frac{10a}{2.37}\right) = P\left(Z \leq \frac{10a}{2.37}\right) - (1 - P\left(Z \leq \frac{10a}{2.37}\right)) = 2P\left(Z \leq \frac{10a}{2.37}\right) - 1$. So $P\left(Z \leq \frac{10a}{2.37}\right) = 0.95$. Thus $\frac{10a}{2.37} = 1.645$. So we get $a = (2.37)(1.645)/10 = 0.3899$.

4a. Use X_1, \dots, X_5 to denote the weights of the encyclopedias and Y_1, \dots, Y_{20} to denote the weights of the novels, so we compute $P(X_1 + \dots + X_5 + Y_1 + \dots + Y_{20} \leq 60) = P\left(\frac{X_1+\dots+X_5+Y_1+\dots+Y_{20}-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \leq \frac{60-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}}\right) = P(Z \leq 0.89) = 0.8133$.

4b. We compute $P(58 \leq X_1 + \dots + X_5 + Y_1 + \dots + Y_{20} \leq 62) = P\left(\frac{58-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \leq \frac{X_1+\dots+X_5+Y_1+\dots+Y_{20}-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}} \leq \frac{62-(5)(6)-(20)(1.4)}{\sqrt{(5)(0.8)^2+(20)(0.3)^2}}\right) = P(0 \leq Z \leq 1.79) = 0.9633 - 0.5000 = 0.4633$.