STAT/MA 41600
In-Class Problem Set #37: November 13, 2015
Solutions by Mark Daniel Ward

1a. The random variable X is a Gamma random variable with $r = 500$ and $\lambda = \frac{1}{2.5}$.

1b. We have $P(X \leq 1200) = \int_0^{1200} \frac{(1/2.5)^{500}}{499!} x^{499} e^{-x/2.5}\,dx$.

1c. We compute $P(X \leq 1200) = P\big(\frac{X-500(2.5)}{\sqrt{500(2.5)^2}} \leq \frac{1200-500(2.5)}{\sqrt{500(2.5)^2}}\big) \approx P(Z \leq -0.89) = P(Z \geq 0.89) = 1 - P(Z < 0.89) = 1 - 0.8133 = 0.1867$.

2a. Let Y denote the weight of such a stone. The probability that such a stone is "big" is $P(Y \geq 80) = P\big(\frac{Y-70}{8} \geq \frac{80-70}{8}\big) = P(Z \geq 1.25) = 1 - P(Z < 1.25) = 1 - 0.8944 = 0.1056$. Therefore, X is a Binomial random variable with $n = 5000$ and $p = 0.1056$.

2b. We have $P(X \leq 500) = \sum_{x=0}^{500} \binom{5000}{x}(0.1056)^x(1-0.1056)^{5000-x}$.

2c. We have $P(X \leq 500) = P(X \leq 500.5) = P\big(\frac{X-5000(0.1056)}{\sqrt{5000(0.1056)(0.8944)}} \leq \frac{500.5-5000(0.1056)}{\sqrt{5000(0.1056)(0.8944)}}\big) \approx P(Z \leq -1.27) = P(Z \geq 1.27) = 1 - P(Z < 1.27) = 1 - 0.8980 = 0.1020$.

3a. We have $P(X < Y) = \sum_{x=0}^{\infty} \frac{(e^{-5000})(5000^x)}{x!} \sum_{y=x+1}^{\infty} \frac{(e^{-4900})(4900^y)}{y!}$.

3b. We have $P(X < Y) = P(X - Y < 0) = P(X - Y < -0.5) = P\big(\frac{X-Y-(5000-4900)}{\sqrt{5000+4900}} \leq \frac{-0.5-(5000-4900)}{\sqrt{5000+4900}}\big) \approx P(Z \leq -1.01) = P(Z \geq 1.01) = 1 - P(Z < 1.01) = 1 - 0.8438 = 0.1562$.

4. We write $U_1, \ldots, U_{300}$ for these Continuous Uniform random variables. Then we have $P(U_1 + \cdots + U_{300} > 1600) = P\big(\frac{U_1+\cdots+U_{300}-300(5)}{\sqrt{300(25/3)}} > \frac{1600-300(5)}{\sqrt{300(25/3)}}\big) = P(Z > 2) = 1 - P(Z \leq 2) = 1 - 0.9772 = 0.0228$.