

STAT/MA 41600  
In-Class Problem Set #39 part 2: November 23, 2015

1. Consider two six sided dice. One die has 2 red, 2 green, and 2 blue sides. The other die has 3 red sides and 3 blue sides. Roll both dice, and let  $X$  denote the number of red sides that appear. Find the variance of  $X$ .
2. A bag of candy contains 10 green M&M's and 10 red M&M's. Suppose that 10 students pick 2 candies each, without replacement. Let  $X$  denote the number of students who get one red and one green candy. Find  $\text{Var}(X)$ .
3. Consider a pair of random variables  $X, Y$  with constant joint density on the triangle with vertices at  $(0, 0)$ ,  $(5, 0)$ , and  $(0, 5)$ . Find the covariance of  $X$  and  $Y$ .
4. Suppose  $X$  and  $Y$  have joint density  $f_{X,Y}(x, y) = e^{1-x}$  for  $x, y$  in the region where  $0 < x < y < 1$ , and  $f_{X,Y}(x, y) = 0$  otherwise. Find the covariance of  $X$  and  $Y$ .

[Note: It might look strange to have a joint probability density function of  $X$  and  $Y$  with no  $y$ 's in it, but this is OK. This function is constant with regard to  $y$ , i.e., it does not change as  $y$  changes. You can check, for instance, that  $f_{X,Y}(x, y)$  is a valid probability density function because it is nonnegative and because  $\int_0^1 \int_x^1 e^{1-x} dy dx = 1$ .]

[Hint: Just to save you having to do so many integration by parts, for your convenience, we have:  $\int_0^1 e^{-x} dx = 1 - e^{-1}$  and  $\int_0^1 xe^{-x} dx = 1 - 2e^{-1}$  and  $\int_0^1 x^2e^{-x} dx = 2 - 5e^{-1}$  and  $\int_0^1 x^3e^{-x} dx = 6 - 16e^{-1}$ .]