STAT/MA 41600
In-Class Problem Set #41: December 2, 2015
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1a. If X denotes the grade, then $P(X \geq 0.95) \leq \frac{\mathbb{E}(X)}{0.95} = \frac{0.80}{0.95} = 0.84$.
1b. We have $P(0.73 \leq X \leq 0.87) = P(|X - 0.80| \leq 0.07) = P(|X - 0.80| \leq k\sigma_X)$ where $\sigma_X = 0.05$ is the standard deviation, and $k = 0.07/0.05$. So we have $P(0.73 \leq X \leq 0.87) \geq 1 - \frac{1}{(0.07/0.05)^2} = 0.49$.

2a. We have $P(|X - 22| \geq 0.5) = P(|X - 22| \geq k\sigma_X)$ where $\sigma_X = 0.3$ and $k = 0.5/0.3$. So we get $P(|X - 22| \geq 0.5) \leq \frac{1}{(0.5/0.3)^2} = 0.36$.

2b. We have $P(X \geq 24) \leq \frac{22}{24} = 0.92$, by the Markov inequality.

2c. We have $P(X \geq 21) \leq \frac{22}{21} = 1.05$, but of course we automatically have an even better bound (without using the Markov inequality), namely, $P(X \geq 21) \leq 1$. So the Markov inequality does not give us any additional information in this case.

3a. We use X for the number of bees. Then we get $P(X \geq 20) \leq \frac{15}{20} = 0.75$.

3b. We have $P(10 \leq X \leq 20) = P(|X - 10| \leq 5) = P(|X - 10| \leq k\sigma_X)$ where $\sigma_X = 3$ and $k = 5/3$. So we get $P(10 \leq X \leq 20) \geq 1 - \frac{1}{(5/3)^2} = 0.64$.

4a. We use X to denote the number of customers. Then $P(20 \leq X \leq 40) = P(|X - 30| \leq 10) = P(|X - 30| \leq k\sigma_X)$ where $\sigma_X = 5$ and $k = 10/5 = 2$. So we conclude $P(20 \leq X \leq 40) \geq 1 - \frac{1}{2^2} = 3/4$.
4b. We have $P(X \geq 40) \leq \frac{30}{40} = 3/4 = 0.75$.
4c. We have $P(X \geq 50) \leq \frac{30}{50} = 3/5 = 0.60$.
4d. We have $P(X \geq 60) \leq \frac{30}{60} = 1/2 = 0.50$.