

STAT/MA 41600  
In-Class Problem Set #42: December 4, 2015

1. Consider three independent continuous Uniform random variables, each of which has a constant density on  $[0, 10]$ .
  - a. Find the density  $f_{X_{(1)}}(x_1)$  of the 1st order statistic (i.e., find the density of the min).
  - b. Find the density  $f_{X_{(2)}}(x_2)$  of the 2nd order statistic.
  - c. Find the density  $f_{X_{(3)}}(x_3)$  of the 3rd order statistic (i.e., find the density of the max).
2. Same setup as question #1.
  - a. Find  $\mathbb{E}(X_{(1)})$ .
  - b. Find  $\mathbb{E}(X_{(2)})$ .
  - c. Find  $\mathbb{E}(X_{(3)})$ .
  - d. Since the sum of the three random variables and the sum of the three order statistics must be the same (always), then their expected values are the same, i.e.,  $X_1 + X_2 + X_3 = X_{(1)} + X_{(2)} + X_{(3)}$ . So  $\mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_{(1)} + X_{(2)} + X_{(3)})$ . We also know that  $\mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 5 + 5 + 5 = 15$ . Use this to double check your answers to parts a, b, c. Do the answers sum up to 15?
3. Suppose  $X_1, X_2$  are independent continuous random variables with  $f_{X_1, X_2}(x_1, x_2) = (1/8)^2(4 - x_1)(4 - x_2)$  on the square  $0 < x_1 < 4$  and  $0 < x_2 < 4$ , and  $f_{X_1, X_2}(x_1, x_2) = 0$  otherwise.
  - a. Find the density  $f_{X_{(1)}}(x_1)$  of the 1st order statistic (i.e., find the density of the min).
  - b. Find the density  $f_{X_{(2)}}(x_2)$  of the 2nd order statistic (i.e., find the density of the max).
4. Same setup as question #3.
  - a. Find  $\mathbb{E}(X_{(1)})$ .
  - b. Find  $\mathbb{E}(X_{(2)})$ .
  - c. Since the sum of the two random variables and the sum of the two order statistics must be the same (always), then their expected values are the same, i.e.,  $X_1 + X_2 = X_{(1)} + X_{(2)}$ . So  $\mathbb{E}(X_1 + X_2) = \mathbb{E}(X_{(1)} + X_{(2)})$ . We also know that  $\mathbb{E}(X_1 + X_2) = \mathbb{E}(X_1) + \mathbb{E}(X_2) = 4/3 + 4/3 = 8/3$ . Use this to double check your answers to parts a, b. Do the answers sum up to  $8/3$ ?