STAT/MA 41600
In-Class Problem Set #44: December 9, 2015

1. Children are decorating rocks with paint and sparkly material, to give as gifts. The weights of the rocks are assumed to be uniformly distributed between 0.5 and 2.2 pounds. Let X denote the weight of such a rock. Suppose that the cost of the materials to be used on such a rock is $Y = (2/5)X + 0.1$.

a. Find the probability density function $f_Y(y)$ of Y. Be sure to specify where $f_Y(y)$ is nonzero.

b. Use $f_Y(y)$ to find the probability that Y is less than 0.60.

c. Check your answer by using $f_X(x)$ to find the probability that $(2/5)X + 0.1$ is less than 0.60.

2. Same setup as #1.

a. What are the mean and standard deviation of the cost Y of the materials used on such a rock?

b. Now suppose that 100 such rocks are to be decorated, and their weights are independent. Use X_j to denote the weight of the jth rock. Thus, the cost of materials used to decorate the jth rock is $Y_j = (2/5)X_j + 0.1$. Find a good approximation for the distribution of the total cost, namely, $Y_1 + \cdots + Y_{100}$.

3. Suppose that X is a continuous random variable that is uniformly distributed on the interval $(0, 3)$. Suppose that we define $Y = (X + 3)(X - 3)$.

a. What is the probability density function $f_Y(y)$ of Y? For which values of y is the density nonzero?

b. Use $f_Y(y)$ to get the mean of Y, as $\mathbb{E}(Y) = \int_{-\infty}^{\infty} y\, f_Y(y)\, dy$.

c. Use $f_X(x)$ to get the mean of Y indirectly, as $\mathbb{E}(Y) = \int_{-\infty}^{\infty} (x + 3)(x - 3)\, f_X(x)\, dx$. Your solution should agree with **3b**.

4. Suppose that the joint distribution of X and Y is uniform in the triangular region of the (x, y)-plane with corners at the origin and $(5, 0)$ and $(5, 2)$.

a. Find $\mathbb{E}(X)$.

b. Find $\mathbb{E}(Y)$.

c. Find $\mathbb{E}(XY)$.

d. Use your solutions to parts a, b, c to find the covariance of X and Y.