1. Children are decorating rocks with paint and sparkly material, to give as gifts. The weights of the rocks are assumed to be uniformly distributed between 0.5 and 2.2 pounds. Let $X$ denote the weight of such a rock. Suppose that the cost of the materials to be used on such a rock is $Y = \frac{2}{5}X + 0.1$.
   a. Find the probability density function $f_Y(y)$ of $Y$. Be sure to specify where $f_Y(y)$ is nonzero.
   b. Use $f_Y(y)$ to find the probability that $Y$ is less than 0.60.
   c. Check your answer by using $f_X(x)$ to find the probability that $(2/5)X + 0.1$ is less than 0.60.

2. Same setup as #1.
   a. What are the mean and standard deviation of the cost $Y$ of the materials used on such a rock?
   b. Now suppose that 100 such rocks are to be decorated, and their weights are independent. Use $X_j$ to denote the weight of the $j$th rock. Thus, the cost of materials used to decorate the $j$th rock is $Y_j = \frac{2}{5}X_j + 0.1$. Find a good approximation for the distribution of the total cost, namely, $Y_1 + \cdots + Y_{100}$.

3. Suppose that $X$ is a continuous random variable that is uniformly distributed on the interval $(0, 3)$. Suppose that we define $Y = (X + 3)(X - 3)$.
   a. What is the probability density function $f_Y(y)$ of $Y$? For which values of $y$ is the density nonzero?
   b. Use $f_Y(y)$ to get the mean of $Y$, as $\mathbb{E}(Y) = \int_{-\infty}^{\infty} y f_Y(y) \, dy$.
   c. Use $f_X(x)$ to get the mean of $Y$ indirectly, as $\mathbb{E}(Y) = \int_{-\infty}^{\infty} (x + 3)(x - 3) f_X(x) \, dx$. Your solution should agree with 3b.

4. Suppose that the joint distribution of $X$ and $Y$ is uniform in the triangular region of the $(x, y)$-plane with corners at the origin and $(5, 0)$ and $(5, 2)$.
   a. Find $\mathbb{E}(X)$.
   b. Find $\mathbb{E}(Y)$.
   c. Find $\mathbb{E}(XY)$.
   d. Use your solutions to parts a, b, c to find the covariance of $X$ and $Y$. 