STAT/MA 41600
Midterm Exam #2: November 18, 2015

Name ____

Purdue student ID (10 digits) ____

1. The testing booklet contains 6 questions, but students only need to answer 4 or 5 of the questions. The 4 or 5 questions chosen by the student will all be weighted evenly (i.e., each question is worth 1/4 or 1/5 of the midterm exam grade, respectively).

2. Permitted Texas Instruments calculators:
   BA-35
   BA II Plus*
   BA II Plus Professional Edition*
   TI-30XS MultiView*
   TI-30Xa
   TI-30XIIS*
   TI-30XIIB*
   TI-30XB MultiView*
   *The memory of the calculator should be cleared at the start of the exam.

3. **Circle your final answer in your booklet**; otherwise, no credit may be given.

4. There is no penalty for guessing or partial work.

5. Show all your work in the exam booklet. If the majority of questions are answered correctly, but insufficient work is given, the exam could be considered for academic misconduct. Therefore, you should *show all your work and justify your solutions* in the exam booklet.

6. Extra sheets of paper are available from the proctor.

A Gamma random variable with parameters λ and r has probability density function:

$$f_X(x) = \begin{cases} \frac{\lambda^r}{(r-1)!} x^{r-1} e^{-\lambda x}, & \text{for } x > 0, \\ 0 & \text{otherwise,} \end{cases}$$

and the cumulative distribution function (CDF) is:

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} \sum_{j=0}^{r-1} \frac{(\lambda x)^j}{j!}, & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

1. Suppose X, Y has joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{36}(3-x)(4-y) & \text{if } 0 \le x \le 3 \text{ and } 0 \le y \le 4, \\ 0 & \text{otherwise.} \end{cases}$$

1a. Find $P(Y > X)$.

1b. Are X and Y independent? Why?

1c. Find the probability density function $f_X(x)$ of X.

Grade this question? Yes No

2. Suppose X and Y are independent Exponential random variables with $\mathbb{E}(X) = 1/2$ and $\mathbb{E}(Y) = 1/5$.

Compute $P(Y > X)$.

Compute $P(Y \leq 4X)$.

Grade this question? Yes No

3. Let X, Y have joint probability density function $f_{X,Y}(x, y) = 18e^{-2x-7y}$ for $0 < y < x$; and $f_{X,Y}(x, y) = 0$ otherwise.

3a. For $y > 0$, find the conditional probability density $f_{X|Y}(x \mid y)$ of X, given $Y = y$.

3b. Find $\mathbb{E}(Y)$.

Grade this question? Yes No

4. Suppose X and Y have a constant joint density on the square with vertices $(0, 0)$, $(0, 3)$, $(3, 3)$, $(3, 0)$. Find $\mathbb{E}(\min(X, Y))$.

Grade this question? Yes No

5. Let $X = X_1 + \cdots + X_{175}$ where each X_i is an Exponential random variable with $\mathbb{E}(X_i) = 2$. Let $Y = Y_1 + \cdots + Y_{120}$ where each Y_j is an Exponential random variable with $\mathbb{E}(Y_j) = 3$. Suppose that all of the X_i's and Y_j's are independent.

Find a good approximation for $P(X < Y)$.

Grade this question? Yes No

6. Suppose that $U_1, \ldots, U_{80}$ are independent, continuous random variables, each of which is Uniformly distributed on the interval $[0, 5]$.
6a. Find a good approximation for $P(192 < U_1 + \cdots + U_{80} < 208)$.
6b. Find a good approximation for $P(|U_1 + \cdots + U_{80} - 200| \leq 15)$.

Grade this question? Yes No