

STAT/MA 41600  
In-Class Problem Set #1: August 24, 2016

1. Consider a collection of 3 dice. One die is red, one die is green, and one die is blue. Each die is 6-sided and is numbered from 1 to 6.

1a. If we roll all 3 dice (1 time each), and we write down the results, keeping track of the 3 values that appear, as well as the corresponding colors, how many possible outcomes are in the sample space  $S$ ? (An outcome is an ordered 3-tuple, e.g., it tells us the values of  $R$  for the red die,  $G$  for the green die, and  $B$  for the blue die.)

1b. How many outcomes have *three distinct values* for  $R$ ,  $G$ , and  $B$ , i.e., all different values?

1c. How many outcomes have  $R < G < B$ ?

1d. How many outcomes have  $G < R < B$ ?

1e. How many outcomes have  $R = G < B$ ?

2. Consider a standard deck of 52 cards. Deal 5 cards in a row, left to right, on the table. So an outcome is an ordered 5-tuple of cards.

2a. How many outcomes are there altogether?

2b. In how many of the outcomes is the leftmost card a Jack?

2c. In how many of the outcomes is the rightmost card a Jack?

2d. In how many of the outcomes is there at least one Jack, among the middle three cards on the table, i.e., among the 2nd, 3rd, and 4th cards?

3. Consider 5 consecutive tosses of a coin.

3a. How many outcomes are there?

3b. How many events are there?

3c. How many outcomes have a head on the last flip (the other four flips can be arbitrary)?

3d. How many events contain the outcome  $(H, H, T, T, T)$ ?

4. Calculus review. Suppose  $\lambda > 0$ . Find a closed form expression for each of these:

4a.  $\int_0^\infty \lambda e^{-\lambda x} dx$

4b.  $\int_a^\infty \lambda e^{-\lambda x} dx$  (assume  $a > 0$  is fixed)

4c.  $\int_0^\infty (x)(\lambda e^{-\lambda x}) dx$

4d.  $\sum_{x=0}^\infty \frac{\lambda^x e^{-\lambda}}{x!}$

4e.  $\sum_{x=3}^\infty \frac{\lambda^x e^{-\lambda}}{x!}$