STAT/MA 41600
In-Class Problem Set #3: August 29, 2016
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Problem Set 3 Answers

1a. The probability is $(.30 + .45)^4 = 81/256 = 0.3164$.
1b. The probability is $.30/(.30 + .45) = 2/5 = 0.40$.

2a. The probability is $12/52 + (40/52)(12/52) + (40/52)^2(12/52) + (40/52)^3(12/52) = 18561/28561 = 0.6499$. Alternatively, this is $1 - (40/52)^4 = 18561/28561 = 0.6499$.
2b. The probability is $(40/52)^4(12/52) = 30000/371293 = 0.0808$.
2c. The probability is
$12/52+(40/52)(12/51)+(40/52)(39/51)(12/50)+(40/52)(39/51)(38/50)(12/49) = 2759/4165 = 0.6624$. Alternatively, this is $1 - (40/52)(39/51)(38/50)(37/49) = 2759/4165 = 0.6624$.
2d. The probability is $(40/52)(39/51)(38/50)(37/49)(12/48) = 703/8330 = 0.0844$.

3. The probability that a role has sum 7 or larger is $1-(1+3+6+10)/216 = 196/216 = 49/54$. The probability that a role has sum exactly 7 is $15/216$, and the probability that the role has sum strictly larger than 7 is $1 - (1 + 3 + 6 + 10 + 15)/216 = 181/216$. Therefore, the desired probability is $(15/216)/(15/216 + 181/216) = 15/(15 + 181) = 15/196 = 0.0765$.

4a. We have $P(A) = 1/2$ and $P(B) = 1/2$ but $P(A \cap B) = 1/3$ so $P(A)P(B) \neq P(A \cap B)$, so A and B are dependent.
4b. We have $P(B) = 1/2$ and $P(C) = 2/3$ but $P(B \cap C) = 1/2$ so $P(B)P(C) \neq P(B \cap C)$, so B and C are dependent. (Alternatively, we could have just noted that B is a nontrivial subset of C.)
4c. We have $P(A) = 1/2$ and $P(C) = 2/3$ and $P(A \cap C) = 1/3$ so $P(A)P(C) = 1/3 = P(A \cap C)$, so A and C are independent.