

**Problem Set 3 Answers**

**1a.** The probability is  $(.30 + .45)^4 = 81/256 = 0.3164$ .

**1b.** The probability is  $.30/(.30 + .45) = 2/5 = 0.40$ .

**2a.** The probability is  $12/52 + (40/52)(12/52) + (40/52)^2(12/52) + (40/52)^3(12/52) = 18561/28561 = 0.6499$ . Alternatively, this is  $1 - (40/52)^4 = 18561/28561 = 0.6499$ .

**2b.** The probability is  $(40/52)^4(12/52) = 30000/371293 = 0.0808$ .

**2c.** The probability is

$12/52 + (40/52)(12/51) + (40/52)(39/51)(12/50) + (40/52)(39/51)(38/50)(12/49) = 2759/4165 = 0.6624$ . Alternatively, this is  $1 - (40/52)(39/51)(38/50)(37/49) = 2759/4165 = 0.6624$ .

**2d.** The probability is  $(40/52)(39/51)(38/50)(37/49)(12/48) = 703/8330 = 0.0844$ .

**3.** The probability that a role has sum 7 or larger is  $1 - (1+3+6+10)/216 = 196/216 = 49/54$ . The probability that a role has sum exactly 7 is  $15/216$ , and the probability that the role has sum strictly larger than 7 is  $1 - (1 + 3 + 6 + 10 + 15)/216 = 181/216$ . Therefore, the desired probability is  $(15/216)/(15/216 + 181/216) = 15/(15 + 181) = 15/196 = 0.0765$ .

**4a.** We have  $P(A) = 1/2$  and  $P(B) = 1/2$  but  $P(A \cap B) = 1/3$  so  $P(A)P(B) \neq P(A \cap B)$ , so  $A$  and  $B$  are dependent.

**4b.** We have  $P(B) = 1/2$  and  $P(C) = 2/3$  but  $P(B \cap C) = 1/2$  so  $P(B)P(C) \neq P(B \cap C)$ , so  $B$  and  $C$  are dependent. (Alternatively, we could have just noted that  $B$  is a nontrivial subset of  $C$ .)

**4c.** We have  $P(A) = 1/2$  and  $P(C) = 2/3$  and  $P(A \cap C) = 1/3$  so  $P(A)P(C) = 1/3 = P(A \cap C)$ , so  $A$  and  $C$  are independent.