STAT/MA 41600
In-Class Problem Set #4: August 31, 2016
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Problem Set 4 Answers

1. Let B denote the event that the values that appear on the two dice are different.
1a. Let A denote the event that the sum is 5. Then $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{4/36}{30/36} = \frac{4}{30} = \frac{2}{15}$.
1b. Let C denote the event of having a 6. Then $P(C \mid B) = \frac{P(C \cap B)}{P(B)} = \frac{10/36}{30/36} = \frac{10}{30} = \frac{1}{3}$.
1c. Let D denote the event that the sum is even. Then $P(D \mid B) = \frac{P(D \cap B)}{P(B)} = \frac{12/36}{30/36} = \frac{12}{30} = \frac{2}{5}$.

2a. The probability is 10/49. The most straightforward way to see this is that, once 3 of the hearts have been used for the 1st, 2nd, and 3rd cards, there are 49 cards remaining, and 10 are hearts, and they are all equally likely to be the 5th card. So the probability is 10/49.
2b. Same reasoning, same answer, works as above.
2c. Let B denote the event that at least 2 of the 5 cards are hearts. Let A denote the event that all 5 cards are hearts. Then $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. Since $A \subset B$, then $A \cap B = A$. So $P(A \mid B) = \frac{P(A)}{P(B)} = \frac{P(A)}{1 - P(B^c)} = \frac{(13/52)(12/51)(11/50)(10/49)(9/48)}{1-(39/52)(38/51)(37/50)(36/49)(35/48)-(5)(13/52)(39/51)(38/50)(37/49)(36/48)} = \frac{33}{24460} = 0.001349$.

3. Let A denote the event that the all-black die is chosen. Let B denote the event that a black side appears. Then $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{(1/2)(1)}{(1/2)(1)+(1/2)(2/6)} = 3/4$.

Alternatively, once a black side appears, it could be any of 8 possible black sides, and all 8 are equally likely, and 6 of them belong to the all-black die. So the probability is $6/8 = 3/4$.

4. We can let 1 red bear sit. Then there are 3 possible places that the other 2 red bears could sit, and they are sitting in those places with probability $(2/8)(1/7)$. Then one bear can sit next to this group (clockwise). Afterwards, the rest of that bear's family is sitting next to that bear with probability $(2/5)(1/4)$. If this happens, then the third family must be sitting together too. So the desired probability is $(3)(2/8)(1/7)(2/5)(1/4) = 3/280$.

Alternatively, if we view the seats as numbered, there are 9! total seating arrangements. There are 9 places where the red bears could sit as a family, which leaves 2 places for the green bears to sit as a family, and just 1 place for the blue bears to sit as a family. Finally, within each family, there are $3! = 6$ possible arrangements. So the desired probability is $(9)(2)(3!)^3/9! = 3/280$.