

STAT/MA 41600  
 In-Class Problem Set #5: September 2, 2016  
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**Problem Set 5 Answers**

**1.** Let  $B$  denote the event that the a red side appears. Let  $A$  denote the event that the first die was chosen. We compute  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A \cap B) + P(A^c \cap B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)} = \frac{(1/2)(2/6)}{(1/2)(2/6) + P(1/2)(1/4)} = 4/7$ .

**2.** Let  $A, I, N$  denote (respectively) the events that the person owns an Android, iPhone, or no cell phone at all. Let  $S$  denote the event that the customer is satisfied.

**2a.** We have  $P(I|S) = \frac{P(I \cap S)}{P(S)} = \frac{P(I \cap S)}{P(I \cap S) + P(A \cap S) + P(N \cap S)} = \frac{P(I)P(S|I)}{P(I)P(S|I) + P(A)P(S|A) + P(N)P(S|N)} = \frac{(0.177)(0.90)}{(0.177)(0.90) + (0.807)(0.70) + (0.016)(0)} = 0.2200$ .

**2b.** We have  $P(N|S^c) = \frac{P(N \cap S^c)}{P(S^c)} = \frac{P(N \cap S^c)}{P(I \cap S^c) + P(A \cap S^c) + P(N \cap S^c)} = \frac{P(I)P(S^c|I)}{P(I)P(S^c|I) + P(A)P(S^c|A) + P(N)P(S^c|N)} = \frac{(0.016)(1)}{(0.177)(0.10) + (0.807)(0.30) + (0.016)(1)} = 0.0580$ .

**2c.** An equivalent question is: What is the probability that the first 3 people are not Android customers? So the desired probability is  $(1 - 0.807)^3 = 0.0072$ .

**3a.** Let  $R$  denote the event that the M&M is red, and  $B$  denotes the probability it is broken. Then  $P(R|B) = \frac{P(R \cap B)}{P(B)} = \frac{(0.13)(0.10)}{(0.24)(0.10) + (0.13)(0.15) + (0.16)(0.10) + (0.20)(0.15) + (0.13)(0.10) + (0.14)(0.10)} = 0.1116$ .

**3b.** Let  $B$  now be the event that the M&M is blue, and  $W$  denotes the probability it is whole. Then  $P(B|W) = \frac{P(B \cap W)}{P(W)} = \frac{(0.24)(0.90)}{(0.24)(0.90) + (0.13)(0.85) + (0.16)(0.90) + (0.20)(0.85) + (0.13)(0.90) + (0.14)(0.90)} = 0.2445$ .

**4.** The probability that Bob takes exactly  $n$  flips is  $(1/2)^{n-1}(1/2) = (1/2)^n$ , and similarly, the probability that Alice gets no heads at all during  $n$  flips of her coin is  $(1/2)^n$ . Let  $B_n$  denote the event that Bob takes exactly  $n$  flips to get heads the first time. Let  $A_n$  be the probability that Alice gets no heads during  $n$  flips of her coin. So the desired probability is  $\sum_{n=1}^{\infty} P(B_n \cap A_n) = \sum_{n=1}^{\infty} (1/4)^n = \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = 1/3$ .