Problem Set 8 Answers

1. We have \( p_X(1) = P(X = 1) = (2/6)(1/4) + (4/6)(3/4) = 7/12, \) and \( p_X(0) = P(X = 0) = (2/6)(3/4) + (4/6)(1/4) = 5/12. \)

2a. We have \( p_X(x) = P(X = x) = (1 - 0.177)^{x-1}(0.177) = (0.823)^{x-1}(0.177) \) for \( x \geq 1, \) and \( p_X(x) = 0 \) otherwise.

2b. We have \( F_X(x) = P(X \leq x) = \sum_{i=1}^{x} (1 - 0.177)^{i-1}(0.177) = 1 - (1 - 0.177)^x = 1 - (0.823)^x \) for \( x \geq 1. \) Alternatively, we compute \( F_X(x) = P(X \leq x) = 1 - P(X > x) = 1 - (0.823)^x \) for \( x \geq 1. \)

2c. We have \( p_Y(y) = P(Y = y) = (1 - (0.177)(0.90))^{y-1}(0.177)(0.90) = (0.8407)^{y-1}(0.1593) \) for \( y \geq 1, \) and \( p_Y(y) = 0 \) otherwise.

3. There are 21 equally likely pairs of outcomes on the dice with sum 7 or larger. So we get \( p_X(7) = 6/21; \) \( p_X(8) = 5/21; \) \( p_X(9) = 4/21; \) \( p_X(10) = 3/21; \) \( p_X(11) = 2/21; \) \( p_X(12) = 1/21. \)

4a. We have \( p_X(1) = 3/5 \) and \( p_X(0) = 2/5. \)

4b. We have \( p_Y(1) = 3/5 \) and \( p_Y(0) = 2/5. \)

4c. The probability mass functions are still the same, even if Bob chooses his cookie first.