STAT/MA 41600
In-Class Problem Set #10: September 14, 2016
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Problem Set 10 Answers

1. We recall from Problem Set #7, question 4, that $P(X = 3) = 2/15$, $P(X = 2) = 1/5$, $P(X = 1) = 2/5$, and $P(X = 0) = 4/15$. Thus $\mathbb{E}(X) = (3)(2/15) + (2)(1/5) + (1)(2/5) + (0)(4/15) = 6/5$.

2a. We recall from Problem Set #7, question 3, that $P(X = 2) = 105/221$, $P(X = 1) = 96/221$, and $P(X = 0) = 20/221$. Thus $\mathbb{E}(X) = (2)(105/221) + (1)(96/221) + (0)(20/221) = 18/13$.

2b. We compute $P(X = 3) = ((36)(35)(34))/((52)(51)(50)) = 21/65$, $P(X = 2) = 3((36)(35)(16))/((52)(51)(50)) = 504/1105$, $P(X = 1) = 3((36)(16)(15))/((52)(51)(50)) = 216/1105$, and $P(X = 0) = ((16)(15)(14))/((52)(51)(50)) = 28/1105$. Thus $\mathbb{E}(X) = (3)(21/65) + (2)(504/1105) + (1)(216/1105) + (0)(28/1105) = 27/13$.

3a. We have $P(X = x) = (4/5)^{x-1}(1/5)$ for $x \geq 1$ and $P(X = x) = 0$ otherwise. Thus we get $\mathbb{E}(X) = \sum_{x=1}^{\infty}(x)(4/5)^{x-1}(1/5) = (1/5) \sum_{x=1}^{\infty} \frac{d}{dq}q^x\Big|_{q=4/5} = (1/5) \frac{d}{dq} \sum_{x=1}^{\infty} q^x\Big|_{q=4/5} = (1/5) \frac{d}{dq}\frac{q}{1-q}\Big|_{q=4/5} = (1/5) \frac{1}{(1-q)^2}\Big|_{q=4/5} = (1/5)\frac{1}{(1-4/5)^2} = 5$.

3b. We have $P(X = x) = 1/5$ for integers $1 \leq x \leq 5$, and $P(X = x) = 0$ otherwise. Therefore we get $\mathbb{E}(X) = (1/5)(1) + (1/5)(2) + (1/5)(3) + (1/5)(4) + (1/5)(5) = 3$.

4a. We have $p_X(6) = 1/6$, $p_X(5) = 1/6$, $p_X(4) = 7/24$, $p_X(3) = 5/24$, $p_X(2) = 3/24$, and $p_X(1) = 1/24$.

4b. We have $\mathbb{E}(X) = (6)(1/6) + (5)(1/6) + (4)(7/24) + (3)(5/24) + (2)(3/24) + (1)(1/24) = 47/12$.