STAT/MA 41600

In-Class Problem Set #12: September 19, 2016

1. Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs.

1a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #7, question 4.

1b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2 + X_3$, where the X_j's are **dependent** indicators. Expand $(X_1 + X_2 + X_3)^2$ into 9 terms, where 6 of them will behave one way, and the other 3 will behave another way.

1c. Find $\text{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.

2. Pick two cards simultaneously at random from a well-shuffled deck of 52 cards. There are 36 cards which have numbers on them (cards 2 through 10, in each of the 4 suits), and there are 16 cards without numbers on them (A, J, Q, K, in each of the 4 suits). Let X be the number of cards that you get with numbers on them.

2a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #7, question 3.

2b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + X_2$, where the X_j's are **dependent** indicators. Expand $(X_1 + X_2)^2$ into 4 terms, where 2 of them will behave one way, and the other 2 will behave another way. Or, if you prefer, write $X = X_1 + \cdots + X_{36}$, where the X_j's are **dependent** indicators. Expand $(X_1 + \cdots + X_3)^2$ into $36^2 = 1296$ terms, where $1296 - 36 = 1260$ of them will behave one way, and the other 36 will behave another way.

2c. Find $\text{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.

3. Reconsider question **2**, but this time pick 3 cards. Find the variance of X.

4. Suppose Alice rolls a 6-sided die, and Bob rolls a 4-sided die. Let X denote the *maximum* value on the two dice.

4a. Find $\mathbb{E}(X^2)$ using the probability mass function of X, as given in Problem Set #10, question 4a.

4b. Find $\mathbb{E}(X^2)$ in a different way, namely, using the fact that $X = X_1 + \cdots + X_6$, where the X_j's are **dependent** indicators. Expand $(X_1 + \cdots + X_6)^2$ into 36 terms, which have various types of behaviors.

Hint: Using the formulation from Problem Set 11, on this problem we used $X_j = 1$ if the maximum is bigger than or equal to j, and $X_j = 0$ otherwise. So on this problem, $X_i X_j = X_j$ if $j > i$. For example, $X_3 X_5 = X_5$ since $X_3 X_5 = 1$ if the maximum is at least 5, and $X_3 X_5 = 0$ otherwise. This problem might seem a little tricky at first, but just think about it, and discuss it, and make sure that your solution agrees with **4a**.

4c. Find $\text{Var}(X)$ using your answer to $\mathbb{E}(X^2)$ and your answer to $\mathbb{E}(X)$ from last week.