STAT/MA 41600
In-Class Problem Set #19: September 30, 2016
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Problem Set 19 Answers

1. The number of Jacks is a Hypergeometric random variable with parameters $N = 52$, $M = 4$, and $n = 5$. So the variance is $940/2873$.

2a. The number of green bears chosen is a Hypergeometric random variable with parameters $N = 9$, $M = 3$, and $n = 5$. So the variance is $5/9$.
2b. The number of bears chosen that are green or blue is a Hypergeometric random variable with parameters $N = 9$, $M = 6$, and $n = 5$. So the variance is again $5/9$.
2c. If X denotes the green bears chosen (as in part **2a**), then the number of green bears that are not chosen is $3 - X$. We compute $\text{Var}(3 - X) = \text{Var}(X) = 5/9$.

3a. Yes, $\mathbb{E}(X)$ and $\mathbb{E}(Y)$ are both equal to $np = nM/N$.
3b. Yes, we have $\frac{N-n}{N-1} \leq 1$. Therefore $\text{Var}\,(X) = n(M/N)(1 - M/N)(N - n)/(N - 1) \leq n(M/N)(1 - M/N) = \text{Var}\,(Y)$.

4a. The number of defective toys checked by the inspector is a Hypergeometric random variable with parameters $N = 50{,}000$, $M = 500$, and $n = 200$. So the probability mass function is $p_X(x) = \frac{\binom{49,500}{200-x}\binom{500}{x}}{\binom{50,000}{200}}$.
4b. The exact expression is $P(X = 4) = p_X(4) = \frac{\binom{49,500}{200-4}\binom{500}{4}}{\binom{50,000}{200}}$.
4c. Since the distribution of X is approximately Binomial with $n = 200$ and $p = 500/50{,}000 = 1/100$, then $P(X = 4) \approx \binom{200}{4}(1/100)^4(99/100)^{196} = 0.0902$.

Alternatively, since the distribution of X is approximately Poisson with $\lambda = np = (200)(1/100) = 2$, then $P(X = 4) \approx e^{-2}2^4 4! = 0.0902$.