STAT/MA 41600

In-Class Problem Set #24: October 12, 2016

1. Consider a random variable X that has a probability density function of the form $f_X(x) = (k)(x)(5-x)$ for $0 \le x \le 5$, and $f_X(x) = 0$ otherwise, where k is a constant.

1a. What is the value of k?

1b. Find the probability that X is bigger than 4 or less than 1, i.e., $P(X > 4 \text{ or } X < 1)$.

2. Suppose that the time needed to wait for the next bus to appear is a random variable Y with probability density function $f_Y(y) = (2/7)e^{-(2/7)(y)}$ for $y > 0$, and $f_Y(y) = 0$ otherwise. Compute the probability that Y is no further than 1 unit away from 3, i.e., $P(|Y - 3| \le 1)$.

3. Suppose that the CDF of a random variable X is $F_X(x) = 1 - e^{-5x}$ for $x > 0$, and $F_X(x) = 0$ otherwise.

3a. What is the probability density function of X?

3b. Use the CDF to compute $P(1/4 < X < 1/3)$. Hint: we have $P(1/4 < X < 1/3) = P(X < 1/3) - P(X \le 1/4)$.

3c. Use the probability density function to find $P(1/4 < X < 1/3)$. Your solution should agree with **3b** (this is just another method of solution).

4. Consider a random variable X whose probability density function is constant on the interval $[30, 100]$, and the pdf is zero otherwise. Compute $P(80 < 2X < 164)$.