

1. Suppose that X and Y have joint probability density function $f_{X,Y}(x,y) = 15e^{-5x-3y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.

1a. Are X and Y independent? Why?

1b. Define $Z = \min(X, Y)$. Find the probability density function of Z .

2. Suppose that X and Y have joint density $f_{X,Y}(x,y) = 24e^{-5x-3y}$ for $y > x > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.

2a. Are X and Y independent? Why?

2b. Calculate $P(Y > 2X)$.

3. Using the joint pdf in 2, find $P(X > 1/10)$.

4. Suppose that X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{12}(4 - xy) & \text{if } 0 < x < 2 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

4a. Are X and Y independent? Why?

4b. Find the probability density function of X .

4c. Can you verify that the pdf you found in 4b is an actual pdf?