

**1.** Suppose that  $X$  and  $Y$  have joint probability density function  $f_{X,Y}(x, y) = 15e^{-5x-3y}$  for  $x > 0$  and  $y > 0$ , and  $f_{X,Y}(x, y) = 0$  otherwise.

**1a.** Is it true in this setup that  $P(Y > 3/10 \mid X > 1/10) = P(Y > 3/10)$ ? Why or why not?

**1b.** Calculate the value of  $P(Y > 3/10 \mid X > 1/10)$ .

**2.** Suppose that  $X$  and  $Y$  have joint density  $f_{X,Y}(x, y) = 24e^{-5x-3y}$  for  $y > x > 0$ , and  $f_{X,Y}(x, y) = 0$  otherwise.

**2a.** Calculate  $P(Y > 3/10 \mid X > 1/10)$ . You may use the fact that  $P(X > 1/10) = e^{-8/10}$ , as calculated in question 3 of the previous problem set.

**2b.** Find  $f_X(x)$ , and use this to find  $f_{Y|X}(y \mid x)$ .

**2c.** Calculate  $P(Y > 3/10 \mid X = 1/10)$ .

**3.** Consider a pair of random variables  $X, Y$  with constant joint density on the triangle with vertices at  $(0, 0)$ ,  $(8, 0)$ , and  $(0, 4)$ .

**3a.** Find  $f_{Y|X}(y \mid 2)$ .

**3b.** Find  $P(Y > 1 \mid X = 2)$ .

**3c.** Find  $P(Y > 1 \mid X > 2)$ .

**4.** Suppose that  $X$  and  $Y$  have joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{12}(4 - xy) & \text{if } 0 < x < 2 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

**4a.** Find  $f_{Y|X}(y \mid x)$ . You are welcome to use  $f_X(x)$ , as calculated in question 4b of the previous problem set.

**4b.** Calculate  $P(Y < 4/3 \mid X = 2/3)$ .

**4c.** Calculate  $P(Y < 4/3 \mid X < 2/3)$ .