

1. Suppose that X and Y have joint probability density function $f_{X,Y}(x,y) = 15e^{-5x-3y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x,y) = 0$ otherwise.

1a. Find $\text{Var}(X)$.

1b. Find $\text{Var}(Y)$.

2. Suppose that X and Y have joint density $f_{X,Y}(x,y) = 24e^{-5x-3y}$ for $y > x > 0$, and $f_{X,Y}(x,y) = 0$ otherwise. Find $\text{Var}(X)$.

3. Consider a pair of random variables X, Y with constant joint density on the triangle with vertices at $(0, 0)$, $(8, 0)$, and $(0, 4)$.

3a. Find $\text{Var}(X)$.

3b. Find $\text{Var}(Y)$.

4. Suppose that X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{12}(4 - xy) & \text{if } 0 < x < 2 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $\text{Var}(X)$.

(Just as an interesting aside, notice that, by symmetry, in this case, $\text{Var}(Y) = \text{Var}(X)$.)