STAT/MA 41600
In-Class Problem Set #31: October 26, 2016
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Problem Set 31 Answers

1. The area of the region where $|X - Y| < 1$ is $100 - (9)(9)/2 - (9)(9)/2 = 19$ (just think about removing the two triangles). So the desired probability is $19/100$.

2. If we let the two students' grades be X and Y, then the pair (X, Y) is uniformly distributed in a 10×10 square, and the desired region, where $X + Y \geq 197$, has area $(3)(3)/2 = 9/2$. So the desired probability is $(9/2)/100 = 9/200$.

3a. The CDF of U is:

$$F_U(u) = \begin{cases} 0 & u < 0, \\ u/5 & 0 \leq u \leq 5, \\ 1 & u > 5. \end{cases}$$

3b. We note that $2 \leq X \leq 17$. Thus, for a in the range $2 \leq a \leq 17$, we have $F_X(a) = P(X \leq a) = P(3U + 2 \leq a) = P(U \leq (a - 2)/3) = ((a - 2)/3)/5 = (a - 2)/15$. So we get

$$F_X(x) = \begin{cases} 0 & x < 2, \\ (x-2)/15 & 2 \leq x \leq 17, \\ 1 & x > 17. \end{cases}$$

3c. From the CDF of X, we see that X is a continuous uniform random variable on the interval $[2, 17]$.

4ab. We know that $0 \leq X \leq 3$. For a in the range $0 \leq a \leq 3$, we have $F_X(a) = P(X \leq a) = P(\max(U, V) \leq a) = P(U \leq a \ \&\ V \leq a) = P(U \leq a)P(V \leq a) = (a/3)(a/3) = a^2/9$. Therefore, we get the CDF and probability density function of X:

$$F_X(x) = \begin{cases} 0 & x < 0, \\ x^2/9 & 0 \leq x \leq 3, \\ 1 & x > 3. \end{cases} \qquad \text{and} \qquad f_X(x) = \begin{cases} 2x/9 & 0 \leq x \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

4cd. We know that $0 \leq Y \leq 3$. For a in the range $0 \leq a \leq 3$, we have $F_Y(a) = P(Y \leq a) = 1 - P(Y > a) = 1 - P(\min(U, V) > a) = 1 - P(U > a \ \&\ V > a) = 1 - P(U > a)P(V > a) = 1 - ((3 - a)/3)((3 - a)/3) = (2/3)a - a^2/9$. Therefore, we get the CDF and probability density function of Y:

$$F_Y(y) = \begin{cases} 0 & y < 0, \\ (2/3)y - y^2/9 & 0 \leq y \leq 3, \\ 1 & y > 3. \end{cases} \qquad \text{and} \qquad f_Y(y) = \begin{cases} 2/3 - 2y/9 & 0 \leq y \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$