

**Problem Set 32 Answers**

**1.** We have  $P(X > U) = \int_0^5 \int_u^\infty (\frac{1}{5})(\frac{1}{2}e^{-x/2}) dx du = \int_0^5 (\frac{1}{5})(e^{-u/2}) du = -\frac{2}{5}e^{-u/2} \Big|_{u=0}^5 = \frac{2}{5} - \frac{2}{5}e^{-5/2}$ .

**2a.** We compute  $P(X > 1.6) = \int_{1.6}^\infty \frac{1}{2.8}e^{-(1/2.8)x} dx = e^{-1.6/2.8} = 0.5647$ .

**2b.** We compute  $P(X > 3.5 \mid X > 1.1) = \frac{P(X > 3.5 \ \& \ X > 1.1)}{P(X > 1.1)} = \frac{P(X > 3.5)}{P(X > 1.1)} = \frac{\int_{3.5}^\infty \frac{1}{2.8}e^{-(1/2.8)x} dx}{\int_{1.1}^\infty \frac{1}{2.8}e^{-(1/2.8)x} dx} = \frac{e^{-3.5/2.8}}{e^{-1.1/2.8}} = e^{-2.4/2.8} = 0.4244$ . Instead, we could have used the memoryless property of exponential random variables, to get  $P(X > 3.5 \mid X > 1.1) = P(X > 2.4) = e^{-2.4/2.8} = 0.4244$ .

**3.** [On the problem set, I originally wrote 15 seconds = 4 minutes, but of course I meant to write 15 seconds = 1/4 minute = 0.25 minutes.] The probability that the next bird is blue is  $\int_0^\infty \int_x^\infty \frac{1}{15}e^{-x/15} \frac{1}{20}e^{-y/20} dy dx = \int_0^\infty \frac{1}{15}e^{-x/15} e^{-x/20} dx = \int_0^\infty (\frac{1}{15})e^{-7x/60} dx = (\frac{1}{15}) \frac{e^{-7x/60}}{-7/60} \Big|_{x=0}^\infty = (\frac{1}{15}) / (\frac{7}{60}) = 4/7$ .

Alternatively, solving the problem with minutes instead of seconds, we get the same answer:  $\int_0^\infty \int_x^\infty 4e^{-4x} e^{-3y} dy dx = \int_0^\infty 4e^{-4x} e^{-3x} dx = \int_0^\infty 4e^{-7x} dx = -(4/7)e^{-7x} \Big|_{x=0}^\infty = 4/7$ .

**4a.** Yes, if  $X$  is an exponential random variable, then  $cX$  is an exponential random variable too. If  $X$  has parameter  $\lambda$ , then  $P(cX > a) = P(X > a/c) = e^{-\lambda(a/c)} = e^{-(\lambda/c)a}$ , so  $\lambda/c$  is the parameter of  $cX$ . Another way to see this is to note that  $\mathbb{E}(X) = 1/\lambda$ , so  $\mathbb{E}(cX) = c/\lambda$ , so that  $\lambda/c$  is the parameter of  $cX$ .

**4b.** No, it is not the case that  $cX + d$  is an exponential random variable. Since  $X$  is always positive, then  $cX + d$  is always bigger than  $d$ , i.e.,  $cX + d$  takes values in the range  $(d, \infty)$ , but exponential random variables take values in the range  $(0, \infty)$ . So  $cX + d$  is not an exponential random variable.