STAT/MA 41600
In-Class Problem Set #36: November 9, 2016
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Problem Set 36 Answers

1a. Let $X_1, \ldots, X_5$ denote the volumes of the containers. Then $P(X_1 + \cdots + X_5 > 10) = P(\frac{X_1+\cdots+X_5-(5)(1.9)}{\sqrt{(5)(0.3)^2}} > \frac{10-(5)(1.9)}{\sqrt{(5)(0.3)^2}}) = P(Z > 0.75) = 1 - P(Z < 0.75) = 1 - 0.7734 = 0.2266$.

1b. We compute $0.90 = P(X_1 + \cdots + X_5 > c) = P(\frac{X_1+\cdots+X_5-(5)(1.9)}{\sqrt{(5)(0.3)^2}} > \frac{c-(5)(1.9)}{\sqrt{(5)(0.3)^2}}) = P(Z > \frac{c-9.5}{0.67}) = P(-Z < -\frac{c-9.5}{0.67})$. Of course Z and $-Z$ have the same distribution, so $0.90 = P(Z < -\frac{c-9.5}{0.67})$, and thus $-\frac{c-9.5}{0.67} = 1.28$, so $c = 8.64$.

2. Let X and Y denote the rainfall for City A and B, respectively. Then $P(X > Y) = P(X - Y > 0) = P(\frac{X-Y-(35-31)}{\sqrt{3^2+2^2}} > \frac{0-(35-31)}{\sqrt{3^2+2^2}}) = P(Z > -1.11) = P(Z < 1.11) = 0.8665$.

3a. Let $X_1, \ldots, X_5$ denote the weights of 5 big rocks. Then $P(X_1 + \cdots + X_5 > 100) = P(\frac{X_1+\cdots+X_5-5(21)}{\sqrt{(5)(2^2)}} > \frac{100-5(21)}{\sqrt{(5)(2^2)}}) = P(Z > -1.12) = P(Z < 1.12) = 0.8686$.

3b. Let $Y_1, \ldots, Y_{1000}$ denote the weights of 1000 small rocks. Then $P(Y_1 + \cdots + Y_{1000} > 10020) = P(\frac{Y_1+\cdots+Y_{1000}-1000(10)}{\sqrt{(1000)(1.5^2)}} > \frac{10020-1000(10)}{\sqrt{(1000)(1.5^2)}}) = P(Z > 0.42) = 1 - P(Z < 0.42) = 1 - 0.6628 = 0.3372$.

4a. Let X be the weight of a big rock, and Y_1 and Y_2 be the weights of two small rocks. Then $P(X > Y_1 + Y_2) = P(X - Y_1 - Y_2 > 0) = P(\frac{X-Y_1-Y_2-(21-10-10)}{\sqrt{2^2+1.5^2+1.5^2}} > \frac{0-(21-10-10)}{\sqrt{2^2+1.5^2+1.5^2}}) = P(Z > -0.34) = P(Z < 0.34) = 0.6331$.

4b. We compute $P(2Y \le X \le 2Y + 1) = P(0 \le X - 2Y \le 1) = P(\frac{0-(21-(2)(10))}{\sqrt{2^2+(2^2)(1.5^2)}} \le \frac{X-2Y-(21-(2)(10))}{\sqrt{2^2+(2^2)(1.5^2)}} \le \frac{1-(21-(2)(10))}{\sqrt{2^2+(2^2)(1.5^2)}}) = P(-0.28 \le Z \le 0) = P(0 \le -Z \le 0.28)$, but Z and $-Z$ have the same distribution, so $P(2Y \le X \le 2Y + 1) = P(0 \le Z \le 0.28) = P(Z \le 0.28) - P(Z \le 0) = 0.6103 - 0.5000 = 0.1103$.