STAT/MA 41600
In-Class Problem Set #43: December 5, 2016

1. If X has probability density function $f_X(x) = 5e^{-5x}$ for $x > 0$, and $f_X(x) = 0$ otherwise, what is the moment generating function $g(t) = M_X(t) = \mathbb{E}(e^{tX})$ of X? (Technical note: You can assume that we will use $t < 5$, since we only need the behavior of $g(t)$ near $t = 0$.)

2a. Find the derivative of the mgf from question **1**, i.e., find $g'(t) = M'_X(t)$.
2b. Use this derivative to find the mean of X, namely, $\mathbb{E}(X) = g'(0)$.
2c. Find the second derivative, namely, $\mathbb{E}(X^2) = g''(0)$.
2d. Does $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$ agree with what you know about the mean and variance of this kind of random variable? (Hint: Note that, once we did integration only 1 time, in question 1, then we never have to do it again to get the mean, variance, etc. In the past, we had to do integration by parts, multiple times, to get the mean and then the variance.)

3. If X has probability mass function $p_X(x) = (4/5)^{x-1}(1/5)$ for integers $x \geq 1$, and $p_X(x) = 0$ otherwise, what is the moment generating function $g(t) = M_X(t) = \mathbb{E}(e^{tX})$ of X? (Technical note: You can assume that we will use $t < \ln(5/4)$, since we only need the behavior of $g(t)$ near $t = 0$.)

4a. Find the derivative of the mgf from question **3**, i.e., find $g'(t) = M'_X(t)$.
4b. Use this derivative to find the mean of X, namely, $\mathbb{E}(X) = g'(0)$.
4c. Find the second derivative, namely, $\mathbb{E}(X^2) = g''(0)$.
4d. Does $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$ agree with what you know about the mean and variance of this kind of random variable? (Hint: Note that, once we did a summation only 1 time, in question 3, then we never have to do it again to get the mean, variance, etc.)