

STAT/MA 41600  
Midterm Exam #1: October 7, 2016

Name \_\_\_\_\_

Purdue student ID (10 digits) \_\_\_\_\_

1. The testing booklet contains 5 questions, but students only need to answer 4 of the questions. The 4 questions chosen by the student will all be weighted evenly (i.e., each question is worth  $1/4$  of the midterm exam grade). The bonus question is *purely optional*.
2. Permitted Texas Instruments calculators:
  - BA-35
  - BA II Plus\*
  - BA II Plus Professional Edition\*
  - TI-30XS MultiView\*
  - TI-30Xa
  - TI-30XIIS\*
  - TI-30XIIB\*
  - TI-30XB MultiView\*

\*The memory of the calculator should be cleared at the start of the exam.
3. **Circle your final answer in your booklet**; otherwise, no credit may be given.
4. There is no penalty for guessing or partial work.
5. Show all your work in the exam booklet. If the majority of questions are answered correctly, but insufficient work is given, the exam could be considered for academic misconduct.
6. Extra sheets of paper are available from the proctor.



1. Students at a certain university are from these disciplines: 15% Science, 21% Healthcare, 24% Liberal Arts, and 40% Engineering. (Each student belongs to a *unique discipline*.) The students attend a lecture and give feedback. Suppose 90% of the Science students liked the lecture, 18% of the Healthcare students liked it, none of the Liberal Arts students liked it, and 10% of the Engineering students liked it. If a student is randomly chosen, and the student liked the lecture, what is the conditional probability that the student is from Science?

Grade this question?    Yes    No

**2.** Alice rolls a 6-sided die until her first 2 appears; let  $X$  denote the number of rolls needed (including the 2 itself).

Bob also rolls a 6-sided die until his first 2 appears; let  $Y$  denote the number of rolls needed (including the 2 itself).

You may assume that  $X$  and  $Y$  are independent.

**2a.** What is  $\text{Var}(X - Y)$ ?

**2b.** What is the probability that the number of Alice's rolls is the same as the number of Bob's rolls?

Grade this question?    Yes    No

**3.** Dr. Ward owns 7 “Deluxe” harmonicas and 12 “Crossover” harmonicas. Without looking at them, they have relatively similar shapes, so he does not notice a difference between them when he reaches into his harmonica container. Suppose that Dr. Ward grabs 8 harmonicas, *without replacement*, and all selections are equally likely.

**3a.** What is the expected number of Deluxe harmonicas that he selects?

**3b.** What is the probability that exactly 5 out of the 8 harmonicas are Deluxe?

**3c.** Reconsider question **3a**, but suppose he replaces each harmonica after selecting it, before selecting the next one. What is the expected number of Deluxe harmonicas that he selects, if he replaces the harmonicas, in between the selections?

Grade this question?    Yes    No

4. Let  $X_1, X_2, X_3, X_4, X_5$  be independent Geometric random variables, with  $\mathbb{E}(X_j) = 3$  for each  $j$ . Define  $X = X_1 + X_2 + X_3 + X_4 + X_5$ . Find  $P(X > 7 \mid X > 5)$ .

Grade this question?    Yes    No

5. Suppose that  $X$  and  $Y$  have joint probability mass function

$$p_{X,Y}(x, y) = (2/3)^x (3/8)^y$$

for positive integers  $x, y$  that satisfy  $1 \leq y \leq x$ .

5a. Find the probability mass function of  $Y$ .

5b. What kind of random variable is  $Y$ ? What is/are the parameter(s)?

Grade this question?    Yes    No

**BONUS (OPTIONAL).** Consider  $n$  families, each consisting of 1 father, 1 mother, and 1 child. The  $3n$  people sit in a circle. A family is happy if all 3 members of the family are sitting in consecutive seats. Find the variance of the number of families that are happy.