1. Let $S$ be the event that the student is from Science, and let $L$ be the event that the student liked the lecture. Then $P(S \mid L) = \frac{P(S \cap L)}{P(L)} = \frac{(15)(.90) + (21)(.18) + (24)(.0) + (40)(.10)}{(15)(.90)} = 0.6344$.

2a. Since $X$ and $Y$ are independent Geometric random variables, each with $p = 1/6$, then $\text{Var}(X - Y) = \text{Var}(X) + (-1)^2 \text{Var}(Y) = \frac{5/6}{(1/6)^2} + \frac{5/6}{(1/6)^2} = 30 + 30 = 60$.

2b. We compute $P(X = Y) = \sum_{n=1}^{\infty} P(X = Y = n) = \sum_{n=1}^{\infty} (5/6)^{n-1}(1/6)(5/6)^{n-1}(1/6) = (1/36) \sum_{n=1}^{\infty} (25/36)^{n-1} = (1/36) \left( \frac{1}{1 - 25/36} \right) = 1/11$.

3a. The number $X$ of Deluxe harmonicas is Hypergeometric with $M = 7$, $N = 19$, and $n = 8$. So $E(X) = (8)(7/19) = 56/19 = 2.9474$. Alternatively, this can be seen by the fact that each harmonica selected has a probability $7/19$ of being a Deluxe harmonica.

3b. Since the number $X$ of Deluxe harmonicas is Hypergeometric with $M = 7$, $N = 19$, and $n = 8$, then $P(X = 5) = \binom{5}{2} \binom{12}{3} / \binom{18}{5} = 770/12597 = 0.0611$.

3c. In this case, the number $X$ of Deluxe harmonicas is Binomial with $n = 8$ and $p = 7/19$. So $E(X) = (8)(7/19) = 56/19 = 2.9474$. Again, this can also be seen by the fact that each harmonica selected has a probability $7/19$ of being a Deluxe harmonica.

4. Since $X$ is Negative Binomial with $r = 5$ and $p = 1/3$, then $P(X > 7 \mid X > 5) = \frac{P(X > 7 \land X > 5)}{P(X > 5)} = \frac{P(X > 7)}{P(X > 5)} = \frac{1 - P(X = 7) - P(X = 6) - P(X = 5)}{1 - P(X = 5)} = \frac{1 - \binom{r}{p}y^r - \binom{r+1}{p}y^{r+1}}{1 - \binom{r}{p}y^r} = \frac{116}{121}$.

5a. We have $p_Y(y) = \sum_{x=y}^{\infty} \binom{r}{p}x^y \left( \frac{1}{p} \right)^y \sum_{x=y}^{\infty} \binom{r}{p}x^y = \binom{r}{p}y^y (1 - \frac{1}{p}) = \binom{r}{p}y^y (1 - \frac{1}{p}) = \binom{r}{p}y^{y-1}$ for $y \geq 1$, and $p_Y(0) = 0$ otherwise.

5b. We note that $Y$ is a geometric random variable with parameter $p = 3/4$.

**Bonus.** If $n = 1$, then the 1 family is always happy, so the variance is 0. Now consider $n \geq 2$. Let $X_j$ denote whether the $j$th family is happy, i.e., $X_j = 1$ if the $j$th family is happy, and $X_j = 0$ otherwise. We compute $E(X_1 + \cdots + X_n) = nE(X_1) = (n)(3)(\frac{2}{3n-1})(\frac{1}{3n-2}) = \frac{6n}{(3n-1)(3n-2)}$, and $E((X_1 + \cdots + X_n)^2) = nE(X_1) + (n^2 - n)E(X_1X_2) = \frac{6n}{(3n-1)(3n-2)} + (n^2 - n)\frac{3n-5}{(3n-1)(3n-2)(n-1)} = \frac{6n}{(3n-4)(3n-1)}$, so we conclude $\text{Var}(X_1 + \cdots + X_n) = \frac{6n}{(3n-4)(3n-1)} - \frac{6n}{(3n-1)(3n-2)^2}$.

Question 1 was like question 2 in problem set 5 from 2015, with only the words changed (but the same numbers).

Question 2 was like questions 3b and 1b in problem set 16 from 2016, with the names of people changed, and the numbers simplified to be a little easier to compute.

Question 3 was like question 2a, 2c, and a comparison to a Binomial, from practice problem set 19.

Question 4 was the very same as question 4a in problem set 17 from 2014.

Question 5 was like question 6c in problem set 9 from 2014, with a small change to the numbers.

The bonus question is motivated by several questions from this semester, for instance, question 4 from problem set 4, and question 1b from problem set 20/22.