STAT/MA 41600
In-Class Problem Set #1: August 23, 2017
Solutions by Mark Daniel Ward

Problem Set 1 Answers

1a. The number of outcomes is $(5)(4) = 20$.
1b. The number of events are $2^{20} = 1{,}048{,}576$.
1c. There are four such outcomes, namely, Diane can choose the 1st, 2nd, 3rd, or 4th seat, and Jack takes the seat to her left.
1d. There are ten such outcomes. There are $\binom{5}{2} = (5)(4)/2 = 10$ ways to pick a pair of seats; we put Diane is the rightmost seat chosen, and put Jack in the leftmost seat chosen.

2a. There are $\binom{4}{2} = (4)(3)/2 = 6$ outcomes.
2b. There are $2^6 = 64$ events.
2c. There is only one outcome not in this event, namely, the outcome that she chooses the blue and green books. So the other 5 possible outcomes are in the event.

Alternative approach to 2abc. If we are unable to distinguish the two red books, then there are fewer possible outcomes, namely, there are 4 possible outcomes (two red books; one red and one blue; one red and one green; one blue and one green), but as we will see in the problem sets in the future, these four outcomes will *not be equally likely to occur*. Using this approach with 4 outcomes, there are $2^4 = 16$ possible events, and there are three outcomes in which there is at least one red book.

3a. There are $6^2 = 36$ outcomes.
3b. There are $2^{36} = 68{,}719{,}476{,}736$ events.
3c. There are 18 outcomes with an even sum.
3d. There are 15 outcomes that have a sum of 8 or larger.

4a. We have $\int_x^\infty 3e^{-3y}\,dy = -e^{-3y}|_{y=x}^\infty = e^{-3x}$.
4b. We have $\int_0^\infty\int_x^\infty (5e^{-5x})(3e^{-3y})\,dy\,dx = \int_0^\infty 5e^{-5x}\int_x^\infty 3e^{-3y}\,dy\,dx$. The inner integral is e^{-3x}, as in part **4a**. So $\int_0^\infty\int_x^\infty (5e^{-5x})(3e^{-3y})\,dy\,dx = \int_0^\infty (5e^{-5x})(e^{-3x})\,dx = \int_0^\infty 5e^{-8x}\,dx = -(5/8)e^{-8x}|_{x=0}^\infty = 5/8$.
4c. We have $\sum_{x=1}^\infty (3/5)^{x-1}(2/5) = (2/5)((3/5)^0 + (3/5)^1 + (3/5)^2 + \cdots) = (2/5)\sum_{x=0}^\infty (3/5)^x = (2/5)(\frac{1}{1-3/5}) = (2/5)/(2/5) = 1$.

4d. This sum is equal to $(3/5)^3$ times the previous sum, so this sum is exactly $(3/5)^3$.
Alternative view: $\sum_{x=4}^\infty (3/5)^{x-1}(2/5) = (2/5)(3/5)^3((3/5)^0 + (3/5)^1 + (3/5)^2 + \cdots) = (2/5)(3/5)^3\sum_{x=0}^\infty (3/5)^x = (2/5)(3/5)^3(\frac{1}{1-3/5}) = (2/5)(3/5)^3/(2/5) = (3/5)^3$.