STAT/MA 41600
In-Class Problem Set #3: August 28, 2017
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Problem Set 3 Answers

1a. On each day, they choose different seats with probability 4/5. So these choose different seats all week with probability $(4/5)^7 = 0.2097$.
1b. On each day, the probability that they both pick seat 1 or 5 is $(2/5)(2/5) = (2/5)^2$. So they both pick seat 1 or 5 throughout the week with probability $(2/5)^{14} = 0.000002684$.
1c. On each day, the probability that there is 1 or more seats between them is 12/25. So these is always one or more seats between, throughout the week, with probability $(12/25)^7 = 0.005871$.

2. There are $\binom{10}{7} = 120$ ways to pick seven distinct numbers and therefore $\binom{10}{7} = 120$ ways to pick a sequence of seven strictly increasing numbers. Each has probability $(1/10)^7$, and all such events are disjoint, so the probability of picking a sequence of 7 strictly increasing numbers during the week is $(120)(1/10)^7 = 0.000012$.

3a. The probability that a sum of 12 occurs before a sum of 9 occurs is $\frac{1/36}{1/36+4/36} = 1/5$. Alternatively, we can compute the probability as $\sum_{j=1}^{\infty}(31/36)^{j-1}(1/36) = \frac{1/36}{1-31/36} = 1/5$.
3b. The probability that a sum of 12 occurs before either die shows a 1 is $\frac{1/36}{1/36+11/36} = 1/12$. Alternatively, we can compute the probability as $\sum_{j=1}^{\infty}(24/36)^{j-1}(1/36) = \frac{1/36}{1-24/36} = 1/12$.

4a. The events A and B are not disjoint, because they have some outcomes in common, such as 14, 16, 18, 20.
4b. Yes, A and B are independent events. To see this, we compute $P(A \cap B) = 4/20 = 1/5$ and $P(A)P(B) = (1/2)(8/20) = 1/5$, so $P(A \cap B) = P(A)P(B)$.
4c. We have $P(A \cup B) = 14/20$, $P(A) = 1/2$, $P(B) = 8/20 = 2/5$, and $P(A \cap B) = 1/5$, so $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ just becomes $14/20 = 1/2 + 2/5 - 1/5$.