STAT/MA 41600

In-Class Problem Set #4: August 30, 2017

1. Consider a standard 52 card deck containing 12 "face cards" (Jacks, Queens, and Kings). Consider a random hand of 5 cards. Let A be the event that all of the cards in the hand are face cards. Let B be the event that at least three of the cards in the hand are face cards.

Find $P(A \mid B)$, i.e., the conditional probability that all 5 cards are face cards, given that at least 3 cards are face cards.

2. Roll five (6-sided) dice. Let D be the event that the sum of the values on the five dice is exactly 8.

Let A be the event that exactly 4 of the 5 dice have the value "1".
Let B be the event that exactly 3 of the 5 dice have the value "1".
Let C be the event that exactly 2 of the 5 dice have the value "1".

2a. Find $P(A \mid D)$.
2b. Find $P(B \mid D)$.
2c. Find $P(C \mid D)$.

[Hint: Given that event D occurs, we know that A or B or C occurs, and these three events are disjoint, so we should have $P(A \mid D) + P(B \mid D) + P(C \mid D) = 1$.]

3a. Flip 7 fair coins. Find the conditional probability that all 7 of them are heads, given that at least 5 of them are heads.
3b. Same question, but using 7 biased coins, which have probability 6/10 of heads and probability 4/10 of tails.

4. Consider a tetrahedron (4-sided die) numbered 1–4, a cube (6-sided die) numbered 1–6, and an octahedron (8-sided die) numbered 1–8. Roll each die one time.
4a. Find the probability that the octahedron's value is both strictly larger than the cube's value and also (simultaneously) strictly larger than the tetrahedron's value.
4b. Find the probability that the octahedron's value is strictly larger than the sum of the cube's value and the tetrahedron's value.