STAT/MA 41600
In-Class Problem Set #4: August 30, 2017
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Problem Set 4 Answers

1. We have $P(A \mid B) = \frac{P(A\cap B)}{P(B)}$ but $A \cap B = A$ in this case, so we get $P(A \mid B) = \frac{P(A)}{P(B)} = \frac{\binom{5}{5}(12/52)(11/51)(10/50)(9/49)(8/48)}{\binom{5}{5}(12/52)(11/51)(10/50)(9/49)(8/48)+\binom{5}{4}(40/52)(12/51)(11/50)(10/49)(9/48)+\binom{5}{3}(40/52)(39/51)(12/50)(11/49)(10/48)}$.

Then we can multiply and divide by (52)(51)(50)(49)(48) on top and bottom, so we get $P(A \mid B) = \frac{95040}{95040+2376000+20592000} = 3/728 = 0.004121$.

2a. We have $P(A \mid D) = \frac{P(A\cap D)}{P(D)}$ but $A \cap D = A$ in this case, so we get $P(A \mid D) = \frac{P(A)}{P(D)} = \frac{\binom{5}{1}P(1,1,1,1,4)}{\binom{5}{1}P(1,1,1,1,4)+\binom{5}{1}\binom{4}{1}P(1,1,1,2,3)+\binom{5}{3}P(1,1,2,2,2)} = \frac{\binom{5}{1}}{\binom{5}{1}+\binom{5}{1}\binom{4}{1}+\binom{5}{3}} = 1/7$.

2b. We have $P(B \mid D) = \frac{P(B\cap D)}{P(D)}$ but $B \cap D = B$ in this case, so we get $P(B \mid D) = \frac{P(B)}{P(D)} = \frac{\binom{5}{1}\binom{4}{1}P(1,1,1,2,3)}{\binom{5}{1}P(1,1,1,1,4)+\binom{5}{1}\binom{4}{1}P(1,1,1,2,3)+\binom{5}{3}P(1,1,2,2,2)} = \frac{\binom{5}{1}\binom{4}{1}}{\binom{5}{1}+\binom{5}{1}\binom{4}{1}+\binom{5}{3}} = 4/7$.

2c. We have $P(C \mid D) = \frac{P(C\cap D)}{P(D)}$ but $C \cap D = C$ in this case, so we get $P(C \mid D) = \frac{P(C)}{P(D)} = \frac{\binom{5}{3}P(1,1,2,2,2)}{\binom{5}{1}P(1,1,1,1,4)+\binom{5}{1}\binom{4}{1}P(1,1,1,2,3)+\binom{5}{3}P(1,1,2,2,2)} = \frac{\binom{5}{3}}{\binom{5}{1}+\binom{5}{1}\binom{4}{1}+\binom{5}{3}} = 2/7$.

3a. We use A and "B" to denote the event of having 7 heads or at least 5 heads, respectively. We get $P(A \mid B) = \frac{\binom{7}{7}(1/2)^7}{\binom{7}{7}(1/2)^7+\binom{7}{6}(1/2)^7+\binom{7}{5}(1/2)^7} = \frac{\binom{7}{7}}{\binom{7}{7}+\binom{7}{6}+\binom{7}{5}} = \frac{1}{1+7+21} = 1/29$.

3b. Same setup, but now we get $P(A \mid B) = \frac{\binom{7}{7}(6/10)^7}{\binom{7}{7}(6/10)^7+\binom{7}{6}(6/10)^6(4/10)+\binom{7}{5}(6/10)^5(4/10)^2} = 1/15$.

4a. There are $4 \times 6 \times 8 = 192$ equally likely outcomes. Let A be the event that the octahedron is strictly largest. Then:

Event A has $4 \times 6 = 24$ outcomes in which the octahedron has value 8.
Event A has $4 \times 6 = 24$ outcomes in which the octahedron has value 7.
Event A has $4 \times 5 = 20$ outcomes in which the octahedron has value 6.
Event A has $4 \times 4 = 16$ outcomes in which the octahedron has value 5.
Event A has $3 \times 3 = 9$ outcomes in which the octahedron has value 4.
Event A has $2 \times 2 = 4$ outcomes in which the octahedron has value 3.
Event A has $1 \times 1 = 1$ outcomes in which the octahedron has value 2.
So A has exactly $24+24+20+16+9+4+1 = 98$ outcomes. So $P(A) = 98/192 = 49/96 = 0.5104$.

4b. There are $4 \times 6 \times 8 = 192$ equally likely outcomes. Let B be the event that the octahedron is strictly larger than the sum of the cube and the tetrahedron. Then:

Event B has 18 outcomes in which the octahedron has value 8.
Event B has 14 outcomes in which the octahedron has value 7.
Event B has 10 outcomes in which the octahedron has value 6.
Event B has 6 outcomes in which the octahedron has value 5.
Event B has 3 outcomes in which the octahedron has value 4.
Event B has 1 outcomes in which the octahedron has value 3.
So B has exactly $18+14+10+6+3+1 = 52$ outcomes. So $P(B) = 52/192 = 13/48 = 0.2708$.