STAT/MA 41600
In-Class Problem Set #7: September 6, 2017
Solutions by Mark Daniel Ward

Problem Set 7 Answers

1. The number of raindrops is a nonnegative integer, so X is a discrete random variable. The time until the first of the raindrops falls is a nonnegative real number, so Y is a continuous random variable.

2. There are 24 equally likely outcomes. We can just count the outcomes corresponding to each possible value of X. We get $P(X = 0) = 4/24 = 1/6$, $P(X = 1) = 7/24$, $P(X = 2) = 6/24 = 1/4$, $P(X = 3) = 4/24 = 1/6$, $P(X = 4) = 2/24 = 1/12$, and $P(X = 5) = 1/24$.

3a. We have $P(X = 1) = 1/6$, $P(X = 2) = (5/6)(1/6)$, $P(X = 3) = (5/6)^2(1/6)$, and $P(X = 4) = (5/6)^3(1/6)$.

3b. We have $P(X > 4) = (5/6)^4$, since $X > 4$ if and only if the first four rolls do not have any occurrences of "3".

Alternatively, we could come $P(X > 4) = 1 - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4)$, which also yields $P(X > 4) = (5/6)^4$.

3c. We have $P(X > n) = (5/6)^n$, since $X > n$ if and only if the first n rolls do not have any occurrences of "3".

3d. For each outcome in which the value "3" is never rolled, it does not make sense for X to have a finite value. On the other hand, the probability of never rolling a "3" is $\lim_{n\to\infty}(5/6)^n = 0$, so the event that a "3" is never rolled can be safely ignored.

4. We write six terms for each probability, according to (respectively) whether Alice gets a 1, 2, 3, 4, 5, or 6.

$P(X = 0) = (1/6)(1/2) + (1/6)(1/2)^2 + (1/6)(1/2)^3 + (1/6)(1/2)^4 + (1/6)(1/2)^5 + (1/6)(1/2)^6 = 21/128$,

$P(X = 1) = (1/6)(1/2) + (1/6)\binom{2}{1}(1/2)^2 + (1/6)\binom{3}{1}(1/2)^3 + (1/6)\binom{4}{1}(1/2)^4 + (1/6)\binom{5}{1}(1/2)^5 + (1/6)\binom{6}{1}(1/2)^6 = 5/16$,

$P(X = 2) = (1/6)(0) + (1/6)\binom{2}{2}(1/2)^2 + (1/6)\binom{3}{2}(1/2)^3 + (1/6)\binom{4}{2}(1/2)^4 + (1/6)\binom{5}{2}(1/2)^5 + (1/6)\binom{6}{2}(1/2)^6 = 33/128$,

$P(X = 3) = (1/6)(0)+(1/6)(0)+(1/6)\binom{3}{3}(1/2)^3+(1/6)\binom{4}{3}(1/2)^4+(1/6)\binom{5}{3}(1/2)^5+(1/6)\binom{6}{3}(1/2)^6 = 1/6$,

$P(X = 4) = (1/6)(0) + (1/6)(0) + (1/6)(0) + (1/6)\binom{4}{4}(1/2)^4 + (1/6)\binom{5}{4}(1/2)^5 + (1/6)\binom{6}{4}(1/2)^6 = 29/384$,

$P(X = 5) = (1/6)(0) + (1/6)(0) + (1/6)(0) + (1/6)(0) + (1/6)\binom{5}{5}(1/2)^5 + (1/6)\binom{6}{5}(1/2)^6 = 1/48$,

$P(X = 6) = (1/6)(0) + (1/6)(0) + (1/6)(0) + (1/6)(0) + (1/6)(0) + (1/6)\binom{6}{6}(1/2)^6 = 1/384$,