STAT/MA 41600

In-Class Problem Set #9: September 11, 2017

1. Suppose that Leo rolls a die until the first appearance of "3" and then stops; let X denote the number of times that he rolls the die. Similarly, suppose that Melissa tosses a fair coin until the first "Head" and then stops; let Y denote the number of times that she flips the coin. Find $P(X = Y)$.

2. Suppose X and Y have joint probability mass function $p_{X,Y}(x, y) = \frac{x}{7y}$ for $1 \leq x \leq y \leq 4$, and $p_{X,Y}(x, y) = 0$ otherwise.

2a. Convince yourself that this is a joint probability mass function.

2b. Find the probability mass function $p_X(x)$.

2c. Find the probability mass function $p_Y(y)$.

3. Roll three dice. Let $X = 1$ if there is at least one occurrence of a "4"; let $X = 0$ otherwise. Let $Y = 1$ if there is at least one occurrence of a "5"; let $Y = 0$ otherwise.

3a. Find $p_{X,Y}(0, 0)$.

3b. Find $p_{X,Y}(0, 1)$.

3c. Find $p_{X,Y}(1, 0)$.

3d. Find $p_{X,Y}(1, 1)$.

[Hint: These are the only four nonzero values for the probability mass function. Do your four values sum to 1 ?]

3e. Find $p_{X|Y}(0|0) = P(X = 0 \mid Y = 0)$ and $p_{X|Y}(1|0) = P(X = 1 \mid Y = 0)$. [Hint: These should add to 1.]

3f. Find $p_{X|Y}(0|1) = P(X = 0 \mid Y = 1)$ and $p_{X|Y}(1|1) = P(X = 1 \mid Y = 1)$. [Hint: These should add to 1.]

4. Roll a blue four-sided die and a red four-sided die. Let Y denote the maximum of the two values that appear. Let X denote the value of the blue die. Find the conditional probability mass function $p_{X|Y}(x \mid y)$ of X given Y. [Hint: There should be 10 conditional probabilities altogether, since you only need to compute $p_{X|Y}(x \mid y)$ for values $1 \leq x \leq y \leq 4$.]